1947-06

The effect of water injection on the educational gas turbine.

Tugend, Robert E.
Rensselaer Polytechnic Institute

http://hdl.handle.net/10945/6393

THE EFFECT OF WATER INJECTION ON
THE EDUCATIONAL GAS TURBINE

ROBERT E. TUGEND

Boston 184

[illegible]

THE EFFECT OF WATER INJECTION ON THE EDUCATIONAL GAS TURBINE

By

Robert E. Tugend

Lieutenant-Commander,

U. S. Navy

Submitted to the Faculty of

Rensselaer Polytechnic Institute

in Partial Fulfilment of the

Requirements for the

Degree of Master of Science

June, 1947

Troy, New York

Table of Contents

Table of Illustrations

Acknowledgments

The author wishes to express his sincere appreciation to Professor Neil P. Bailey, Head of the Department of Mechanical Engineering, Rensselaer Polytechnic Institute, for the suggestion of the topic of this paper as one possible method of increasing flexibility of gas turbines and for his help and practical suggestions throughout the preparation of this paper. Also, the author appreciates the cooperation of the Commanding Officer, Naval Supply Depot, Scotia, New York, by his generous loan of the calculating machine on which the work of this paper was performed and without which, the work would have been lengthened considerably.

The author also expresses gratitude to his wife, Patty, who repeatedly stood many hours watch to prevent the baby's crying from interfering with the completion of this paper within the prescribed time limit.

Definition of Symbols

Area - square inches	A
Specific Heat at Constant Pressure Btu/lb. °F	C_p
Specific Heat at Constant Volume Btu/lb. °F	C_v
Gravitational Constant 32.18 ft/sec/sec.	g
Lower heating value of fuel 19100 Btu/lb.	H
Enthalpy Btu/lb.	h
Conversion factor 778 ft-lbs/Btu	J
Ratio of specific heats	k
Ratio of specific heats for Combustion Gases	k_g
Compressor work Btu	L_c
Net work	L_n
Turbine work Btu	L_T
Logarithm to the base e	ln
Machine Number	M
Thermal efficiency percent	n
Compressor efficiency percent	n_c
Turbine efficiency percent	n_T
Static pressure pounds/square inch	P
Total pressure pounds/square inch	P_o
Gas constant lbs.ft/°R	R
Gas Constant for Combustion Gases lbs.ft/°R	R_g
Entropy for constant pressure processes Btu/lb °F	S_p
Entropy for constant temp. processes Btu/lb °F	S_T
Static temperature °R	T

Total temperature ^oR	T_o
Ideal total temperature	T_o'
Volume cubic feet	V
Velocity at turbine nozzle throat ft/sec	v_n
Turbine wheel blade speed ft/sec	W
Weight of fuel lbs.	w
Weight of Compressor air lbs.	W_a
Weight of Bleed off air lbs.	W_b
Water-fuel ratio lbs. water/lb.fuel	x

The Effect of Water Injection on the Educational Gas Turbine

I. Introduction

During the course of study at Rensselaer Polytechnic Institute in the gas turbine field, one of the major limitations of gas turbine application was discussed very frequently. That limitation was the fact that the gas turbine has a very limited range of operation due to the very small margin between the gross work of the turbine and the gross work of the compressor. This margin, or net work, increases as turbine inlet temperatures increase, but the maximum temperatures at the turbine inlet are limited by available materials.

Several suggestions for increasing the net work of a gas turbine cycle have been put forth. They include increasing the machine efficiencies of the turbines and compressors by better design, increase of turbine inlet temperatures by use of better materials for burners and turbines, use of wet compression to increase the density of the working medium and to increase compressor efficiency, and the use of water injection in the combustion chamber as an effective means of lowering maximum combustor and turbine

temperatures and as a means of increasing the turbine work output without increasing the work of compression.

In January 1947, the Mechanical Engineering Department of Rensselaer Polytechnic Institute received an Educational Gas Turbine from the Aircraft Gas Turbine division of the General Electric Company at West Lynn, Massachusetts. This gas turbine is a type B22 turbo supercharger equipped with a combustion chamber, compressor inlet flow nozzle venture, compressor discharge bleed off and accessory equipment for use in the study of gas turbine performance. A complete description of the unit is contained in reference 1.

The unit, as constructed, has a very low thermal efficiency, extremely small operating range and a small output of bleed off air. Although the primary purpose of the unit is for educational purposes connected with the study of gas turbines, it is hoped that the machine can also be used to augment the supply of compressed air which is used to a considerable extent in the laboratory in which this gas turbine is installed.

Since the turbine has already been fabricated and is at present writing being installed, it appears that the simplest method of altering it to increase the output of bleed off air is the installation of a water injection system in the combustion chamber.

The purpose of this paper is to predict the performance of the Educational Gas Turbine when such a water injection system is installed.

II. Main Text

A diagram showing the main components of the gas turbine with a water injection system, together with the reference points used in the computations is shown in Figure 10.

The compressor and turbine data used in the calculations are based on the curves of Estimated Compressor Performance and curves of Typical Turbine Efficiencies, Figure 1 and Figure 2, which were obtained from reference 1. The values of enthalpy and entropy for the various gases of combustion are obtained from Hecks Gas Tables in reference 2 and from reference 3. The values of R (gas constant) were obtained from reference 4. The values of C_p for the computation of the ratio of specific heats of the gases of combustion are obtained from figures 16 through 20. Figure 16 through figure 20 were obtained from reference 5. The C_p used in each case is obtained from the experimental curve on the graph and not from the straight line approximation of the curve. It is to be noted that the value of C_p obtained is in units of Btu per pound mole degree Fahrenheit. In order to use the values in the computations, they must be divided by the molecular weight of the gas.

All computations are based on a fuel composition of 85 per cent carbon and 15 per cent hydrogen with a lower heating value of 19100 B.t.u. per pound. Dry atmospheric air at 70°F (530°R) is assumed at the entrance of the compressor. The water injected in the combustion chamber is assumed to be in the liquid state at 70°F and to be pure so as to prevent mineral deposits from forming in the machinery. The work necessary to operate all auxiliaries such as the water pump for water injection and the blower for cooling air to the turbine has been neglected since the power source for these auxiliaries comes from an outside source rather than from the gross work of the turbine. The pressure drop through the burner and piping have been neglected making

$$\frac{P_{o2}}{P_{o1}} = \frac{P_{o5}}{P_{o6}}$$

At present, there have been no measurements of the actual pressure loss from the compressor to the turbine, so it was decided that calculations would be made on the basis that these losses are equal to zero. This assumption results in an over prediction of results, in that the net work output as calculated is higher than what can actually be expected from the unit. Experimental data on performance will thus modify the results obtained in this paper slightly. It is not expected, however, that experimental data will modify the basic conclusions drawn from this paper.

The calculation for thermal efficiency is based on the assumption that the machine using the bleed off air is 100% efficient. Any losses that occur in the use of the bleed off air should be properly charged to the machine using the air.

Calculations:

The detailed calculations for each compressor pressure ratio and each burner temperature chosen appear in the appendices. Each appendix shows the set of calculations for one pressure ratio-temperature combination using the water fuel ratio (x) as the variable. In each set of calculations the water-fuel ratio was increased from 0 to the maximum amount of water per pound of fuel that could be used and still obtain perfect combustion. Thus the maximum water fuel ratio varies from 7 with a burner temperature of 1960° R to 9 when the burner temperature is 1660°R.

In order to determine the turbine work, it is necessary to determine Cp and kg for each burner temperature and each combination of the gases of combustion. The calculations for enthalpy changes are made using data from Hecks Gas Tables in reference 2 and from reference 3. The calculations for C_p and k_g were made using data from figure 16 through figure 20.

In order to check the accuracy of the method of obtaining C_p and k_g using basic data from two sources, a

special calculation was made in Appendix 1 using data from Hecks Gas tables only. The comparison of the two methods shows that the value of k_g obtained using the two sources of data is smaller than the k_g which results from the use of Hecks Gas Tables exclusively. But these different values of k_g do not alter the value of the turbine work. Examination of Col. 62 and Col. 95 in Appendix 1 shows that the turbine work obtained from each method of calculation do not vary significantly. Therefore, since the calculation of C_p and k_g are considerably simpler using the data on Figure 16 through Figure 20, this method of calculation is used in all other appendices where it is necessary to evaluate k_g.

Since the value of k varies with the initial temperature of an expansion process and is practically independent of the pressure ratio of the expansion, it is necessary to evaluate k_g for each burner temperature, but once evaluated, it can be used with any pressure ratio provided the burner temperature remains constant. Values of k_g for each burner temperature are plotted on Figure 6.

III. Sample Calculation

To determine the amount of bleed off air and the thermal efficiency of the Educational Gas Turbine at point 1 on Figure 1 when the burner temperature is 1960 °R. Assume a water fuel ratio of 1.0

From Figure 1 we see that the compressor pressure ratio is 2.20, the compressor temperature rise factor is .35 and the compressor speed is 19660 R.P.M.

Lower heating value of fuel = 19100 Btu/lb.

Fuel composition is 85% carbon and 15% hydrogen.

Temperature inlet air = $530^\circ R$.

Temperature rise factor = .35 = $\dfrac{T_{o2} - T_{o1}}{T_{o1}}$

$$\therefore T_{o2} = 530^\circ + .35\,(530) = 715.5^\circ R$$

Using a basic heat balance:

The heat input of the fuel minus the heat adsorbed by the products of combustion equals the heat gained by the air.

or

$$\begin{aligned} wH &- \text{weight of carbon } (h_{1960} - h_{715.5}) \\ &- \text{weight of hydrogen } (h_{1960} - h_{715.5}) \\ &- \text{weight of injected water } (h_{1960} - h_{715.5}) \\ &= W_a\,(h_{1960} - h_{715.5}) \quad \ldots\ldots\ldots\ldots (1) \end{aligned}$$

assume w = 1 lb.

then:

$$19100 - .85\,(\Delta h) - .15\,(\Delta h) - x\,(\Delta h) = W_a\,(\Delta h) \quad \ldots\ldots (2)$$

Using the values of Δh for CO_2, H, H_2O and air obtained from ref. 2 and ref. 3, the equation becomes

$$\begin{aligned} 19100 &- .85\,(374.16 - 37.60) - .15\,(4978.2 - 604.6) - 1763.5x \\ &= (365.0 - 42.25)\,W_a \quad \ldots\ldots\ldots\ldots (3) \end{aligned}$$

$$\text{or } 18157 - 1763.5x = (322.7)\,W_a. \quad \ldots\ldots\ldots\ldots (4)$$

An equation similar to equation (4) appears at the beginning of each appendix for the determination of W_a

Now assuming that $x = 1 \frac{\text{lb. water}}{\text{lb. fuel}}$

equation (4) becomes

$18157 - 1763.5 = (322.7)\ W_a$ . . . . (5)

or $W_a = 50.80$ lbs. air per lb. fuel

In order to find the weight of O_2, N_2, CO_2 and H_2O in the products of combustion, it is first necessary to find the amount of O_2 required for perfect combustion of the fuel.

or $(\frac{32}{12} \times .85) + (\frac{16}{2} \times .15) =$ lbs of O_2 required for perfect combustion . . . . (6)

or $2.266 + 1.200 = 3.466$ lbs O_2 required . . . . . . (7)

The amount of O_2 required for perfect combustion remains constant in all calculations and appears as a constant in the appendices.

The composition of air by weight is about 23% O_2 and 77% N_2, therefore the total O_2 available is $50.80 \times .23 = 11.68$ lbs. O_2 . . . . (8)

The amount of free O_2 in the products of combustion equals the O_2 available minus the O_2 required for combustion of the fuel, or

$11.68 - 3.466 = 8.21$ lbs. free O_2 . . . . . (9)

The amount of N_2 = (50.80)(.77) = 39.12 lbs. . . (10)

The amount of $CO_2 = \frac{44}{12}(.85) = 3.116$ lbs. . . . . (11)

The amount of $H_2O = \frac{18}{2}(.15) + x = 1.35 + 1 = 2.35$ lbs. (12)

The amount of CO_2 and H_2O formed by the combustion of the fuel remain constant for all calculations and appear as constants in the appendices.

The addition of the weights of O_2, N_2, CO_2 and H_2O in the combustion products gives the total weight of the combustion gases. The total weight of the combustion gases is also equal to the weight of fuel (always assumed to be 1 lb.) plus the water-fuel ratio plus the weight of air.

For this case

Weight of combustion products = 8.21 + 39.12 + 3.116 + 2.35

= 52.80 lbs. . . . . . (13)

Now proceed to find a weighted R_g for the combustion gases taking into account the actual composition of the gas mixture.

From ref. 4

$R_{O_2} = 48.31$

$R_{N_2} = 55.16$

$R_{CO_2} = 35.13$

$R_{H_2O} = 85.81$

$$R_g = \frac{W_{O_2}(R_{O_2}) + W_{N_2}(R_{N_2}) + W_{CO_2}(R_{CO_2}) + W_{H_2O}(R_{H_2O})}{W_{O_2} + W_{N_2} + W_{CO_2} + W_{H_2O}} \quad . . . (14)$$

O_2	8.21 (48.31)	=	396.6
N_2	39.12 (55.16)	=	2157.9
CO_2	3.116 (35.13)	=	109.6
H_2O	2.35 (85.81)	=	201.6
	Total	=	2865.7

and $R_g = \dfrac{2865.7}{52.80} = 54.27$

Now find the weighted C_p for the combustion gases taking into account the actual composition of the gas mixture.

From figures 16 through 20, at 1960 R

C_{pO_2} = .2641 Btu/lb.

C_{pN_2} = .2829 Btu/lb.

C_{pCO_2} = .3002 Btu/lb.

C_{pH_2O} = .5622 Btu/lb.

and similar to equation (14)

$$C_p = \frac{W_{O_2}(C_{pO_2}) + W_{N_2}(C_{pN_2}) + W_{CO_2}(C_{pCO_2}) + W_{H_2O}(C_{pH_2O})}{W_{O_2} + W_{N_2} + W_{CO_2} + W_{H_2O}} \quad . \; . \; (15)$$

O_2	8.21 (.2641)	=	2.1683
N_2	39.12 (.2829)	=	11.0631
CO_2	3.116 (.3002)	=	.9354
H_2O	2.35 (.5622)	=	1.3212
	Total	=	15.4880

and $C_p = \dfrac{15.4880}{52.80} = .2933$

Now find k_g for the combustion gases

$$C_p = \left(\frac{k}{k-1}\right)\frac{R}{J} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

$$\text{or } k = \frac{\frac{J}{R}\ C_p}{\frac{J}{R}\ C_p - 1} \quad \ldots\ldots\ldots\ldots\ldots \quad (17)$$

Using the values of R_g and C_p found above

$$k_g = \frac{\frac{778\ (.2933)}{54.27}}{\frac{778\ (.2933)}{54.27} - 1} = \frac{4.206}{3.206} = 1.312$$

In order to enter figure 2 to obtain a turbine efficiency, it is first necessary to obtain the ratio W/v_n

By measurement, the average turbine blade radius is 5.62 inches or .468 feet. To convert the blade speed in RPM to ft/sec. use the formula:

$$W = 2\pi r \times \frac{R.P.M.}{60} \quad \ldots\ldots\ldots\ldots\ldots \quad (18)$$

in this case $W = 2\pi R \left(\frac{19660}{60}\right) = 963$ ft/sec.

Columns 54 and 55 in Appendix 1 show the conversion of turbine blade speed in R.P.M. to feet per second and the results are plotted on figure 3 so that the conversion may be picked off the graph.

From page 5, chapter 1 of reference 6

$$\frac{v_n}{\sqrt{k_g R T_o}} = \frac{M}{\sqrt{1 + \frac{k-1}{2} M^2}} \quad \ldots\ldots \quad (19)$$

$$\text{or} \quad v_n = M\sqrt{\frac{k_g g R_g T_{o5}}{1 + \frac{k_g - 1}{2} M^2}} \quad \ldots\ldots \quad (20)$$

To determine the critical pressure ratio of the turbine nozzle, use the equation relating Mach Number with pressure ratio from page 6, chapter 1 of reference 6.

$$\frac{P_o}{P} = \left[\frac{k-1}{2}\left(M^2 + \frac{2}{k-1}\right)\right]^{\frac{k}{k-1}} \quad \ldots\ldots\ldots \quad (21)$$

Substituting M = 1.0 and the value of k obtained above (k = 1.312) and solving equation (21), the critical pressure ratio is found

$$\frac{P_o}{P} = 1.83$$

Since $\frac{P_{o5}}{P_{o6}} = 2.20$ is greater than the critical pressure ratio, we may use M = 1.0 in equation 20. Equation 20 then becomes

$$v_n = \sqrt{\frac{k_g R_g (32.17)(1960)}{1 + \frac{k_g - 1}{2}}} = \sqrt{\frac{k_g R_g \ 63053}{1 + \frac{k_g - 1}{2}}} \quad \ldots (22)$$

An equation similar to equation (22) appears at the beginning of each appendix for the determination of v_n.

Using the values of k_g and R_g as previously determined and substituting them in equation (22),

$$v_n = \frac{4,489,500}{1.156} = 1971 \text{ ft/sec.}$$

therefore

$$\frac{W}{v_n} = \frac{963}{1971} = .488 \quad \ldots\ldots\ldots\ldots \quad (23)$$

Entering figure 2 with the arguments $\frac{P_{o5}}{P_{o6}} = 2.20$ and

$\frac{W}{V_n} = .488$, the turbine efficiency is determined to be 63.3%

To determine the ideal temperature of the combustion gases leaving the turbine, use the equation for reversible expansion

$$\frac{T_{o5}}{T_{o'6}} = \left(\frac{P_{o5}}{P_{o6}}\right)^{\frac{k-1}{k}} \ldots\ldots\ldots\ldots\ldots (24)$$

Using $\frac{P_{o5}}{P_{o6}} = 2.20$, $T_{o5} = 1960^{\circ}R$ and $k = 1.312$ in equation (24)

$T_{o'6} = 1625^{\circ}R.$

By definition $n_T = \frac{T_{o5} - T_{o6}}{T_{o5} - T_{o'6}} \ldots\ldots\ldots\ldots\ldots (25)$

or $\Delta T_o = n_T\,(\Delta T_o') \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$

Using $n_T = 63.3$, $\Delta T_o' = 1960 - 1625$

$\Delta T_o = 63.3\ (335) = 212^{\circ}$

From the general energy equation

$dL = C_p dT \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$

or

$$L_T = \int_{T_{o6}}^{T_{o5}} C_p dT = C_p \Delta T_o \ldots\ldots\ldots\ldots (28)$$

Using $C_p = .2933$ and $\Delta T_o = 212$

$L_T = .2933\ (212) = 62.2$ Btu/lb combustion gas

The ratio of lbs. of combustion gas to lbs. of inlet or compressor air is $\frac{52.80}{50.80} = 1.040$

Multiplying the turbine work by the ratio of combustion products to inlet air gives

$$L_T = 62.2\ (1.040) = 64.7\ \text{Btu/lb. inlet air}$$

From equation (27), the compression work equals

$$L_C = \int_{T_{o1}}^{T_{o2}} C_p dT \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (29)$$

In this example

$$L_C = h_{715.5} - h_{530} = 44.6\ \text{Btu/lb.}$$

The net work of the gas turbine cycle is

$$W_b L_n = \left[L_T(W_a - W_b) - L_C(W_a)\right] W_b \; . \; . \; . \; . \; . \; . \; . \; . \; (30)$$

An overall work balance gives

$$(W_a)\ L_C + W_b L_n = (W_a - W_b)\ L_T \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (31)$$

Combining equation (31) with equation (30)

$$L_C(W_a) + \left[L_T(W_a - W_b) - L_C(W_a)\right] W_b = L_T(W_a - W_b) \; ... \; (32)$$

Assume $W_a = 1$ lb./sec. then

$$L_C + \left[L_T(1 - W_b) - L_C\right] W_b = L_T\ (1 - W_b) \; . \; . \; . \; . \; . \; . \; . \; . \; (33)$$

or using $L_C = 44.6$ Btu/lb.

$$44.6 + \left[L_T(1 - W_b) - 44.6\right] W_b = L_T(1 - W_b) \; . \; . \; . \; . \; . \; . \; (34)$$

or simplifying

$$L_T(W_b)^2 + \left[44.60 - 2L_T\right] W_b + L_T - 44.60 = 0 \; . \; . \; . \; . \; . \; . \; (35)$$

At the beginning of each appendix there appears a formula similar to equation (35) for the determination of W_b .

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Equation (35) is a quadratic equation in W_b.
To solve it, use the regular quadratic solution

$$W_b = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \ldots\ldots\ldots\ldots \quad (36)$$

let $a = L_T$ let $b = L_C - 2L_T$

let $C = L_T - L_C$

For this example

$a = 64.7$ $b = -84.6$ and $c = 20.0$

Placing these values in equation 36

$$W_b = \frac{84.6 \pm 44.6}{129.2}$$

Using the plus sign we obtain the impossible answer that $W_b = 1$ lb. air/lb. inlet air

Using the negative sign

$W_b = .309$ lbs. air/lb. inlet or compressor air. Answer.

$$\text{Thermal efficiency} = \frac{\text{net heat output}}{\text{net heat input}} \times 100$$

$$\text{or } n = \frac{W_b \times W_a \times L_C \times 100}{19100} \quad \ldots\ldots\ldots\ldots\ldots \quad (37)$$

Using $W_a = 50.80$ lbs. air/lb. fuel, $W_b = .309$ lbs.air/lb. inlet air, $L_C = 44.60$

$$n = \frac{.309\ (50.80)\ (44.60)\ 100}{19100} = 3.67\% \text{ Answer}$$

An equation similar to equation (37) appears at the beginning of each appendix for the determination of n.

All work in the appendices is similar to that in the sample calculation above except that in the first part of Appendix 1, as mentioned before, the values of k_g and C_p are calculated from Kecks Gas Tables. The method used in this calculation is outlined in the notes at the beginning of the appendix. Another exception is that in Appendix B the pressure ratio $\frac{P_{o5}}{P_{o6}}$ is less than the critical pressure ratio for the combustion gases through the turbine nozzle and M can not be assumed equal to 1 as in the sample calculation. The method of procedure in this case is outlined in the notes at the beginning of Appendix B.

IV. <u>Results of Calculations</u>

Calculations were made for the following combinations of $\frac{P_{o5}}{P_{o6}}$ and T_{o5} for points 1, 2, and 3 on figure 1:

$\frac{P_{o5}}{P_{o6}}$	T_{o5}		
2.20	1960°R	-	Appendix 1
2.00	1960°R	-	Appendix A
1.80	1960°R	-	Appendix B
2.20	1860°R	-	Appendix C
2.20	1760°R	-	Appendix D
2.20	1660°R	-	Appendix E

Figure 5 shows the comparison of the net output (W_b) when burner temperature T_{o5} is kept constant at 1960°R and the pressure ratio is varied from 1.80 to 2.20. It is well to note at this point that pressure ratio can be varied by changing the speed of the gas turbine by varying the amount of bleed off air and the amount of fuel injected. From Figure 5, the amount of bleed off air increases as the water fuel ratio increases. With a water fuel ratio of 7, the output increases approximately 100% over that obtained without the use of water injection.

Figure 7 shows the net output when the pressure ratio is kept constant at 2.20 and the burner outlet temperature is varied from 1960°R to 1660°R. This graph shows that at lower temperatures where more water may be injected and still keep perfect combustion of fuel the output of the gas turbine is increased as much as 370%. This graph also shows that by the use of the correct water fuel ratio the output at low burner temperatures can be increased above that obtained by the use of high temperatures alone without water injection.

A comparison of Figures 5 and 7 shows that for any given water fuel ratio, the net output of the cycle can be varied over a wider range by varying the burner temperature than by varying the pressure ratio. From this fact, we see that the turbine is much more sensitive to temperature changes than it is to pressure changes.

Figure 8 shows the cycle or thermal efficiencies attained at a pressure ratio of 2.20 for various burner temperatures. Due to the losses incurred by the latent heat of vaporization of water, it is expected that overall efficiency will drop with increasing amounts of water. This is generally the case, but since the turbine efficiency of this gas turbine is so low (in the vicinity of 62%) that overall efficiency actually increases with water injection at low burner temperatures. From the graph we see that the maximum efficiency with a burner temperature of 1660°R occurs in the vicinity of a water fuel ratio of 6.0. Beyond the water fuel ratio of 6.0, the thermal efficiency at 1660°R actually exceeds that obtained with a burner temperature of 1960°R. Indeed, this graph indicates that by use of water injection the gas turbine could be made to operate at temperatures so low that it wouldn't even idle without the use of water. In order to do this it would be necessary to initially use a high burner temperature, inject a large amount of water, and then gradually reduce temperature to the low temperature desired.

Figure 9 shows the thermal efficiencies for a constant burner temperature and pressure ratios from 1.80 to 2.20. As expected for the high burner temperature, the efficiency decreases steadily as the water fuel ratio is increased. As the pressure ratio is increased, for any water fuel ratio, the thermal efficiency is increased.

V. Recommendations and Conclusion

The above results show that the installation of water injection on the combustion chamber of the Educational Gas Turbine would result in a substantial increase in the output of compressed air from the unit and furthermore that the water injection will increase the overall efficiency of the plant at high pressure ratios and low burner temperatures. It is recommended that this alteration be made.

Obviously, from figure 1, calculations could be made for a great number of combinations of pressure ratio and burner temperatures that were not made in this paper. The author picked the points calculated with the following limitations in mind: The maximum operating speed of the gas turbine is about 21,000 R.P.M., the maximum allowable burner temperature is 1960°R, and the operating range will have to be on the flat portions of the pressure ratio volume flow curves and the compressor efficiency curves. It is recommended that any future calculations made on this turbine be made at lower pressure ratios than those chosen by the author.

List of References

Reference 1 - Bulletin No. DF81523, "Educational Gas Turbine Performance, Installation and Operation," General Electric Company, Aircraft Gas Turbine Division, West Lynn, Massachusetts.

Reference 2 - Principles of Heat Engineering, Neil P. Bailey, John Wiley & Sons, New York, N. Y. 1942.

Reference 3 - Steam Tables and Mollier Diagram, Joseph H. Keenan, 1930 Edition, Lord Baltimore Press, Baltimore, Md.

Reference 4 - Mechanical Engineers Handbook, Lionel S. Marks, Fourth Edition, McGraw-Hill Book Company, New York, N. Y.

Reference 5 - "Gas Turbines with Water Injection," R. H. Zimmerman and C. A. Norman, Paper No. 46-A-47, The American Society of Mechanical Engineers, New York, N. Y.

Reference 6 - "Thermodynamics of High Velocity Flow," Neil P. Bailey, Rensselaer Polytechnic Institute, Sept. 1946.

Appendix I

Tables of calculations assuming a compressor pressure ratio of 2.20 and a burner temperature of 1960°R at point 1 on figure 1. 19660 R.P.M. = 963 ft/sec.

From Fig. 1, compressor temperature rise factor = .35

$T_{o2} = 530 + .35\ (530) = 715.5^{\circ}R$

Using Hecks Gas Tables

$19100 - .85\ (374.16 - 37.60) - .15(4978.2 - 604.6)$

$- 1763.5x - W_a(365.0 - 42.25)$

or $18157 - (1763.5)x - W_a(322.7)$

Columns 1 through 4 are calculated from the above equation. Columns 5 through 9 give the amounts of O_2, N_2, CO_2 and H_2O in the products of combustion. Column 10 gives the total pounds of combustion products per pound of fuel. Columns 11 through 16 show the calculation of S_p for the products of combustion at 1960°R from information in Hecks Gas Tables. Columns 17 through 22 show the calculation of R_g for the gases of combustion. Columns 23 through 34 show the construction of tables of S_p for products of combustion at 1600°R and 1700°R.

$$\Delta S_T = \frac{R_g}{J} \ln \frac{P_{o3}}{P_{o6}}$$

$$\Delta S_T = \frac{R_g}{778} \ln 2.20 = R_g(.0010132)$$

$$S_{p6} = S_{p5} - \Delta S_T$$

Column 35 gives the calculation for ΔS_T

Column 36 gives the calculation for S_{p6}

Columns 37 through 40 show the calculation for T_{o6}'

$$\frac{T_{o5}}{T_{o6}'} = \left(\frac{P_{o5}}{P_{o6}}\right)^{\frac{k-1}{k}}$$

$$\text{or } k = \frac{1}{1 - \dfrac{\ln \dfrac{T_{o5}}{T_{o6}}}{\ln \dfrac{P_{o5}}{P_{o6}}}}$$

Columns 41 through 45 show the calculation for k_g

$$C_{pg} = \frac{k}{k-1}\;\frac{R}{J}$$

Columns 46 through 48 show the calculation for C_p

$$\frac{v_n}{\sqrt{kgRT_o}} = \frac{M}{\sqrt{1 + \frac{k-1}{2} M^2}} \quad , \quad M = 1.0 \text{ at nozzle throat}$$

$$\text{then } v_n = \sqrt{\frac{kR(32.17)\ 1960}{1 + \frac{k-1}{2}}} = \sqrt{\frac{kR_g 63053}{1 + \frac{k-1}{2}}}$$

Columns 49 through 53 show the calculation for v_n

Average turbine blade radius = 5.62 inches or .468 feet.

Turbine blade speed $= 2\pi r \times \frac{\text{r.p.m.}}{60} = .04901 \times \text{r.p.m.}$

Columns 54 and 55 show the calculation for turbine blade speed in feet per second.

Column 56 shows the value of the ratio of wheel speed to nozzle velocity.

Column 57 shows turbine efficiencies as determined from column 56 and Figure 2.

Columns 58 through 62 show the calculation for turbine work in Btu/lb. of compressor air.

$$L_c = \int_{530}^{715.5} C_p dt - 44.6 \text{ Btu/lb. air}$$

An overall heat balance gives:

$$44.60\ W_a + \left[L_T(W_a - W_b) - 44.6\ W_a\right] = L_T(W_a - W_b)$$

or assuming $W_a = 1$ lbs/sec.

$$(\text{Col. } 62)\ (W_b)^2 + \left[44.60 - 2\ (\text{Col.}62)\right] W_b + \text{Col.}62 - 44.60 = 0$$

let a = col. 62 let b = 44.60 - 2 (col.62)

let c = col. 62 - 44.60

$$\text{then } W_b = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Columns 63 through 71 show the calculation for W_b in pounds of air per pound of compressor air.

Columns 72 through 95 show the calculation for C_p, k_g and L_T from information obtained from graphs of C_p for the various products of combustion for purposes of comparison with previously calculated results.

$$\text{Thermal efficiency} = \frac{\text{net heat output}}{\text{net heat input}} \times 100$$

$$= \frac{W_b \times W_a \times L_C \times 100}{19100}$$

$$= \text{col. } 71 \times \text{col. } 4 \times .2335$$

Columns 96 and 97 show the calculations for thermal efficiency.

1	2	3	4	5	6
x	Δh H_2O	18157 - Col. 2	W_a Col. 3 ÷ 322.7	Total O_2 W_a x (.23)	Free O_2 Col. 5 - 3.466
0	0	18157.0	56.26	12.94	9.46
1	1763.5	16393.5	50.80	11.68	8.21
2	3527.0	14630.0	45.34	10.43	6.96
3	5290.5	12866.5	39.87	9.17	5.70
4	7054.0	11103.0	34.40	7.91	4.44
5	8817.5	9339.5	28.94	6.66	3.19
6	10581.0	7576.0	23.48	5.40	1.93
7	12344.5	5812.5	18.01	4.14	0.67
8	14108.0	4049.0	12.55	2.886	---

1	7	8	9	10	11
x	N_2 Col. 4 x .77	CO_2	H_2O Col. 1 + 1.35	Total comb. products Col.6+Col.7 +Col.8+Col.9	Col. 6 x .3080
0	43.32	3.116	1.35	57.26	2.914
1	39.12	3.116	2.35	52.80	2.529
2	34.91	3.116	3.35	48.34	2.144
3	30.70	3.116	4.35	43.87	1.756
4	26.49	3.116	5.35	39.40	1.368
5	22.28	3.116	6.35	34.94	0.9825
6	18.08	3.116	7.35	30.48	0.5944
7	13.87	3.116	8.35	26.01	0.2064

1	12	13	14	15	16
x	Col. 7 x .3345	Col. 8 x .3260	Col. 9 x .6254	Total Col.11 + Col. 12+ Col.13+Col.14	S_{p5} Col.15 ÷ Col.10
0	14.491	1.017	.8443	19.266	.3365
1	13.086	1.017	1.4697	18.102	.3428
2	11.677	1.017	2.0951	16.933	.3503
3	10.269	1.017	2.7205	15.763	.3593
4	8.861	1.017	3.3459	14.592	.3703
5	7.453	1.017	3.9713	13.424	.3842
6	6.048	1.017	4.5967	12.256	.4021
7	4.639	1.017	5.2221	11.084	.4261

1	17	18	19	20	21
x	O_2 Col. 6 x 48.31	N_2 Col. 7 x 55.16	CO_2 Col. 8 x 35.13	H_2O Col. 9 x 85.81	Total Col.17 + Col.18+ Col.19+Col.20
0	457.013	2389.531	109.606	115.843	3071.933
1	396.625	2157.859	109.606	201.653	2865.743
2	336.238	1925.637	109.606	287.463	2658.944
3	275.367	1693.412	109.606	373.273	2451.658
4	214.496	1461.188	109.606	459.083	2244.373
5	154.109	1228.965	109.606	544.893	2037.573
6	93.238	997.293	109.606	630.703	1830.885
7	32.368	765.069	109.606	716.513	1623.556

1	22	23	24	25	26
x	R_g Col. 21 ÷ Col.10	O_2 Col. 6 x .2554	N_2 Col. 7 x .2780	CO_2 Col. 8 x .2664	H_2O Col. 9 x .5151
0	53.65	2.41608	12.0429	0.8312	0.6954
1	54.27	2.09683	10.8754	0.8312	1.2105
2	55.00	1.77758	9.7049	0.8312	1.7256
3	55.89	1.45578	8.5346	0.8312	2.2407
4	56.96	1.13398	7.3642	0.8312	2.7558
5	58.32	0.81473	6.1938	0.8312	3.2709
6	60.08	0.49292	5.0262	0.8312	3.7860
7	62.42	0.17112	3.8559	0.8312	4.3011

1	27	28	29	30	31
x	Total Col.23 + Col.24+ Col.25+Col.26	S_p 1600°R $\frac{\text{Col. 27}}{\text{Col. 10}}$	O_2 Col. 6 x .2710	N_2 Col. 7 x .2947	CO_2 Col. 8 x .2840
0	15.9856	.2792	2.5637	12.7664	0.8861
1	15.0139	.2844	2.2249	11.5267	0.8861
2	14.0393	.2904	1.8862	10.2880	0.8861
3	13.0623	.2978	1.5447	9.0473	0.8861
4	12.0852	.3067	1.2032	7.8067	0.8861
5	11.1106	.3180	0.8645	6.5659	0.8861
6	10.1363	.3326	0.5230	5.3282	0.8861
7	9.1593	.3521	0.1816	4.0875	0.8861

1	32	33	34	35	36
x	H_2O Col. 9 x .5473	Total Col.29 + Col.30 + Col.31+Col.32	S_p 1700°R $\frac{\text{Col. 33}}{\text{Col. 10}}$	ΔS_T Col. 22 x .0010132	S_{p6} Col. 16 - Col. 35
0	0.7389	16.9551	.2961	.0544	.2821
1	1.2862	15.9259	.3016	.0550	.2878
2	1.8335	14.8938	.3081	.0557	.2946
3	2.3808	13.8589	.3159	.0566	.3027
4	2.9281	12.8241	.3255	.0577	.3126
5	3.4754	11.7919	.3375	.0591	.3251
6	4.0027	10.7600	.3530	.0609	.3412
7	4.5700	9.7252	.3739	.0632	.3629

1	37	38	39	40	41	42
x	Col. 34 minus Col. 28	Col. 36 minus Col. 28	$\left(\frac{\text{Col. 38}}{\text{Col. 37}}\right)$ x 100	$T_{o'6}$ Col. 39 + 1600	$\frac{T_{o5}}{T_{o6}}$ $\frac{1960}{\text{Col.40}}$	ln e of $\frac{T_{o5}}{T_{o6}}$
0	.0169	.0029	17.16	1617.2	1.2120	0.1924
1	.0172	.0034	19.77	1619.8	1.2100	0.1906
2	.0177	.0042	23.73	1623.7	1.2071	0.1882
3	.0181	.0049	27.07	1627.1	1.2046	0.1861
4	.0188	.0059	31.38	1631.4	1.2014	0.1835
5	.0195	.0071	36.41	1636.4	1.1978	0.1805
6	.0204	.0086	42.16	1642.2	1.1935	0.1769
7	.0218	.0108	49.54	1649.5	1.1882	0.1725

1	43	44	45	46	47	48
			k		$\frac{R}{J}$	C_{pg}
x	$\frac{\text{Col. 42}}{0.7885}$	1 minus Col. 43	$\frac{1}{\text{Col. 44}}$	$\frac{k}{k-1}$	$\frac{\text{Col. 22}}{778.2}$	Col. 46 x Col.47
0	0.2440	0.7560	1.323	4.096	.06894	0.2824
1	0.2417	0.7583	1.319	4.135	.06974	0.2884
2	0.2387	0.7613	1.314	4.185	.07068	0.2958
3	0.2360	0.7640	1.309	4.236	.07182	0.3042
4	0.2327	0.7673	1.303	4.300	.07319	0.3147
5	0.2289	0.7711	1.297	4.367	.07494	0.3273
6	0.2244	0.7756	1.289	4.460	.07720	0.3443
7	0.2188	0.7812	1.280	4.571	.08021	0.3666

1	49	50	51	52	53
		$k_g R_g$			v_n ft/sec.
x	$1-\frac{k-1}{2}$	Col. 45 x Col.22	Col. 50 x 63053	$\frac{\text{Col. 51}}{\text{Col. 49}}$	$\sqrt{\text{Col. 52}}$
0	1.1615	70.979	4,475,439	3,853,154	1963
1	1.1595	71.582	4,513,460	3,892,592	1973
2	1.1570	72.270	4,556,840	3,938,496	1984
3	1.1545	73.160	4,612,957	3,995,632	1999
4	1.1515	74.219	4,679,731	4,064,030	2016
5	1.1485	75.641	4,769,392	4,152,714	2038
6	1.1445	77.443	4,883,013	4,266,503	2065
7	1.1400	79.898	5,037,809	4,419,131	2102

54 Turbine Speed R.P.M.	55 Turbine Blade Speed ft/sec.
10000	490.1
11000	539.1
12000	588.1
13000	637.1
14000	686.1
15000	735.1
16000	784.2
17000	833.2
18000	882.2
19000	931.2
20000	980.2
21000	1029.2
22000	1078.2
23000	1127.2

1 x	56 W/V_n 963 / Col.53	57 n_T %	58 $T_{o5}-T'_{o6}$ 1960 minus Col.40	59 $T_{o5}-T_{o6}$ act. Col.58 x Col.57	60 L_T Col.59 x Col.48	61 Ratio Comb.prod. / Inlet air Col.10÷Col.4
0	.490	63.2	342.8	216.6	61.2	1.019
1	.488	63.3	340.2	215.3	62.1	1.040
2	.485	63.4	336.3	213.2	63.0	1.065
3	.482	63.5	332.9	211.4	64.3	1.100
4	.478	63.6	328.6	209.0	65.8	1.145
5	.473	63.8	323.6	206.4	67.6	1.207
6	.466	63.9	317.8	203.1	70.0	1.298
7	.458	64.0	310.5	198.7	72.9	1.443

1 x	62 L_T Btu/lb. Air thru turbine Col.60x Col.61	63 2 x Col.62	64 b 44.60 minus Col. 63	65 c Col. 62 minus 44.60	66 b^2 Col. 64 x Col. 64
0	62.3	124.6	-80.0	17.7	6400
1	64.6	129.2	-84.6	20.0	7160
2	67.5	135.0	-90.4	22.9	8170
3	70.7	141.5	-96.9	26.1	9390
4	75.5	151.0	-106.4	30.9	11350
5	81.6	163.0	-118.4	37.0	14050
6	90.8	181.5	-136.9	46.2	18700
7	105.2	210.5	-165.9	60.6	27450

1	67	68	69	70	71
x	4 ac 4 x Col.62 x Col.65	b^2-4ac Col.66-Col.67	$\sqrt{b^2-4ac}$ Sq.root Col.68	minus Col.64 minus Col.69	W_b $\frac{\text{lbs. air}}{\text{lbs.comp.air}}$ $\frac{\text{Col.70}}{\text{Col.63}}$
0	4410	1990	44.6	35.4	.284
1	5170	1990	44.6	40.0	.309
2	6180	1990	44.6	45.8	.340
3	7380	2010	44.8	52.1	.368
4	9330	2020	44.9	61.5	.407
5	12100	1950	44.2	74.2	.455
6	16800	1900	43.6	93.3	.514
7	25500	1950	44.2	121.7	.578

1	72	73	74	75	76
x	O_2 Col.6 x .2641	N_2 Col.7 x .2828	CO_2 Col.8 x .3002	H_2O Col.9 x .5622	Total Col.72 + Col.73 + Col.74 + Col.75
0	2.4984	12.2509	.9354	.7590	16.4437
1	2.1683	11.0631	.9354	1.3212	15.4880
2	1.8381	9.8725	.9354	1.8834	14.5294
3	1.5054	8.6820	.9354	2.4456	13.5684
4	1.1726	7.4914	.9354	3.0078	12.6072
5	.8425	6.3008	.9354	3.5700	11.6487
6	.5097	5.1130	.9354	4.1322	10.6903
7	.1769	3.9224	.9354	4.6944	9.7291

1	77	78	79	80	81
x	C_p Col. 76 / Col. 10	$\frac{J}{R}$ 778 / Col. 22	$\frac{JC_p}{R}$ Col. 78 x Col. 77	$\frac{J}{R}$ C_p-1	k_g Col. 79 / Col. 80
0	.2872	14.50	4.164	3.164	1.316
1	.2933	14.34	4.206	3.206	1.312
2	.3006	14.15	4.253	3.253	1.307
3	.3093	13.92	4.305	3.305	1.303
4	.3200	13.66	4.371	3.371	1.297
5	.3334	13.34	4.448	3.448	1.290
6	.3507	12.95	4.542	3.542	1.282
7	.3741	12.46	4.661	3.661	1.273

1	82	83	84	85	86
X	$1+\frac{k-1}{2}$	k_gR_g Col.81 x Col. 22	k_gR_g63053 Col. 83 x 63053	Col. 84 / Col. 82	v_n ft/sec. $\sqrt{\text{Col. 85}}$
0	1.158	70.603	4,451,731	3,844,327	1961
1	1.156	71.202	4,489,500	3,883,650	1971
2	1.154	71.885	4,532,565	3,927,699	1982
3	1.152	72.825	4,591,835	3,985,967	1996
4	1.149	73.877	4,658,166	4,054,104	2014
5	1.145	75.233	4,743,666	4,142,939	2036
6	1.141	77.023	4,856,531	4,256,381	2063
7	1.137	79.461	5,010,254	4,406,555	2099

1	87	88	89	90	91	92
x	$\frac{W}{V_n}$ $\frac{963}{\text{Col.86}}$	n_T from Fig.2	$\frac{k-1}{k}$ $\frac{\text{Col.81}-1}{\text{Col.81}}$	$2.20^{\frac{k-1}{k}}$	T_{06} $\frac{1960}{\text{Col.90}}$	1960 minus Col.91
0	.491	63.2	.240	1.208	1623	337
1	.488	63.3	.238	1.2065	1625	335
2	.485	63.4	.235	1.2035	1629	331
3	.482	63.5	.232	1.201	1632	328
4	.478	63.6	.229	1.198	1636	324
5	.473	63.8	.225	1.194	1642	318
6	.466	63.9	.220	1.189	1648	312
7	.458	64.0	.214	1.184	1655	305

1	93	94	95	96	97
x	ΔT Col.92 x Col.88	L_T Btu/lb.gas Col.93 x Col.77	L_T Btu/lb. inlet air Col.61 x Col.94	Col.71 x Col.4	n Col.96 x .2335
0	213	61.2	62.4	15.98	3.73
1	212	62.2	64.7	15.70	3.67
2	210	63.1	67.3	15.40	3.595
3	208	64.3	70.7	14.60	3.41
4	206	65.9	75.5	14.00	3.27
5	203	67.7	81.7	13.17	3.065
6	199	69.9	90.6	12.08	2.82
7	195	72.9	105.3	10.41	2.43

Appendix A

Tables of Calculations assuming a compressor pressure ratio of 2.00 and a burner temperature of $1960^{\circ}R$ at point 2 on figure 1. 18500 RPM = 907 ft/sec.

Temperature rise factor = .31

$$T_{o2} = 530 + .31\ (530) = 694^{\circ}R$$

Using Hecks Gas Tables

$$18427 - x(1763.5) = W_a\ (328.0)$$

Columns A1 through A4 show the calculation for the pounds of air per pound of fuel.

Columns A5 through A11 show the calculations for L_T using the applicable figures from Appendix 1.

$$L_c = \int_{530}^{694} C_p dT = 39.38 \text{ Btu/lb. of air.}$$

In the same manner as in Appendix 1, an overall heat balance gives

$$(\text{Col. A11})(W_b)^2 + \left[39.38 - 2(\text{Col. A11})\right] W_b + \text{Col. A11} - 39.38 = 0$$

let a = Col. A11 let b = 39.38 - 2(Col. A11)

let c = Col. A11 - 39.38

and solve for W_b

Columns A12 through A20 show the calculations for W_b

Column A21 shows the calculation for thermal efficiency.

$$\text{Thermal efficiency} = W_a \times W_b \times \frac{39.38 \times 100}{19100}$$

$$= W_a \times W_b \times .206$$

A1 x	A2 Δh H_2O	A3 18142.7 minus Col. 2	A4 W_a Col.3 ÷ 328.0	A5 n_T from Col. A5A and graph	A6 $2.00^{\frac{k-1}{k}}$ from Col. 89
0	0	18142.7	55.31	64.0	1.182
1	1763.5	16379.2	49.94	64.0	1.1793
2	3527.0	14615.7	44.56	64.0	1.177
3	5290.5	12852.2	39.18	64.0	1.1746
4	7054.0	11088.7	33.81	64.0	1.172
5	8817.5	9325.2	28.43	64.0	1.169
6	10581.0	7561.7	23.05	64.0	1.1647
7	12344.5	5798.2	17.68	63.8	1.160

A1 x	A7 $T_{o',6}$ $\frac{1960}{\text{Col. A6}}$	A8 $\Delta T'$ 1960 minus Col. A7	A9 ΔT Col. A8 x Col. A5	A5A $\frac{W}{V}$ $\frac{907}{\text{Col. 53}}$	A10 n_T Btu/lb.gas Col. A9 x Col. 77
0	1658	302	193.0	.462	55.4
1	1662	298	191.0	.460	56.0
2	1666	294	188.0	.457	56.5
3	1669	219	186.0	.454	57.4
4	1672	288	184.0	.450	58.8
5	1676	284	182.0	.445	60.6
6	1681	279	178.3	.439	62.6
7	1690	270	172.5	.432	64.6

A1 x	A11 n_r Btu/lb.air Col. A10 x Col. 61	A12 2 x Col. A11	A13 b = 39.38 minus Col. A12	A14 c = Col. A11 minus 39.38	A15 b^2= Col. A13 x Col. A13	A16 4ac = 4 x Col. A11 x Col. A14
0	56.5	113.0	-73.6	16.4	5210	3660
1	58.3	116.6	-77.2	18.1	5715	4165
2	60.2	120.4	-81.0	20.4	6420	4880
3	63.1	126.2	-86.8	23.6	7500	5950
4	67.4	134.8	-95.4	27.7	8990	7440
5	73.1	146.2	-106.8	33.3	11240	9690
6	81.2	162.4	-123.0	41.8	15120	13600
7	93.3	186.6	-147.2	53.9	21650	20130

A1 x	A17 b^2- 4ac= Col. A15 minus Col. A16	A18 $\sqrt{b^2 - 4ac}$= Sq.root Col. A17	A19 minus Col. A13 minus Col. A18	A20 W_b $\frac{\text{lbs.air}}{\text{lbs.inlet air}}$ = $\frac{\text{Col. A19}}{\text{Col. A12}}$	A21 n .206 x Col. A4 x Col. A20
0	1550	39.4	34.2	.302	3.44
1	1550	39.4	37.8	.324	3.34
2	1540	39.3	41.6	.345	3.17
3	1550	39.4	47.4	.376	3.03
4	1550	39.4	56.0	.416	2.90
5	1550	39.4	67.4	.461	2.70
6	1520	39.1	83.6	.514	2.44
7	1520	39.1	107.8	.577	2.10

Appendix B

Tables of calculations assuming a compressor pressure ratio of 1.80 and a burner temperature of 1960°R at point 3 on Figure 1. 17000 RPM = 834 ft/sec.

Temperature rise factor = .25

$T_{o2} = 530 + .25\ (530) = 662.4\ ^{o}R$

Using Hecks Gas Tables

$18120.3 - x(1763.5) = W_a(335.6)$

Columns B1 through B4 show the calculations for the pounds of air per pound of fuel.

$$M_5 = \sqrt{\frac{2}{k-1}\left[\left(\frac{P_{o5}}{P_{o6}}\right)^{\frac{k-1}{k}} - 1\right]}$$

and $v_n = \dfrac{M_5\sqrt{kgR_gT_{o5}}}{\sqrt{1+\frac{k-1}{2}M_5^2}}$

Since $\frac{P_{o5}}{P_{o6}}$ is less than the critical pressure ratio for the gases of combustion, v_n must be calculated from the Mach. number in the nozzle throat. Columns B5 through B14 show these calculations, using applicable figures from Appendix 1.

Columns B15 through B21 show the calculations for turbine work in Btu per pound of compressor air.

$$L_c = \int_{530}^{662.4} C_p dT = 31.87 \text{ Btu/lb. air.}$$

Setting up a heat balance in the same manner as in appendix 1

$$(\text{Col.B21})(W_b)^2 + \left[31.87 - 2(\text{Col.B21})\right] W_b + \text{Col.B21} - 31.87 = 0$$

let a = Col. B 21 let b = 31.87-2(Col. B 21)

let c = Col. B 21-31.87

and solve for W_b

Columns B22 through B30 show the calculations for W_b

$$n = W_a \times W_b \times \frac{39.38 \times 100}{19100}$$

or $$n = W_a \times W_b \times .167$$

Column B31 shows the thermal efficiency.

B1	B2	B3	B4	B5	B6
x	Δh_{H_2O}	18120.3 minus Col. B2	W_a $\frac{\text{Col. B3}}{335.6}$	$\left(\frac{P_{o5}}{P_{o6}}\right)^{\frac{k-1}{k}}$ from Col. 89	$\left(\frac{P_{o5}}{P_{o6}}\right)^{\frac{k-1}{k}} - 1$ Col. B5 minus 1
0	0	18120.3	53.99	1.1518	.1518
1	1763.5	16356.8	48.74	1.1502	.1502
2	3527.0	14593.3	43.48	1.1482	.1482
3	5290.5	12829.8	38.23	1.1462	.1462
4	7054.0	11066.3	32.97	1.1442	.1442
5	8817.5	9302.8	27.72	1.1417	.1417
6	10581.0	7539.3	22.47	1.1382	.1382
7	12344.5	5775.8	17.21	1.1342	.1342

B1	B7	B8	B9	B10	B11
x	$\frac{2}{k-1}$ from Col. 81	Col. B7 x Col. B6	M_5 Sq. root Col. B8	$\frac{k-1}{2}$ from Col. 82	Col. B10 x Col. B8 + 1
0	6.33	.960	.981	.158	1.1518
1	6.41	.963	.9825	.156	1.1502
2	6.51	.965	.984	.154	1.1487
3	6.60	.966	.986	.152	1.1469
4	6.74	.972	.9875	.149	1.1449
5	6.90	.977	.990	.145	1.1418
6	7.10	.982	.992	.141	1.1385
7	7.33	.985	.994	.137	1.1350

B1	B12	B13	B14	B15	B16
x	$\frac{\text{Col. 51}}{\text{Col. B11}}$	Square root Col.B12	v_n = Col.B13 x Col.B9	$\frac{U}{v_n}$ = $\frac{834}{\text{Col. B14}}$	n_T from Fig.2
0	3,885,604	1971	1933	.431	63.6
1	3,924,065	1981	1948	.428	63.5
2	3,966,954	1992	1960	.426	63.5
3	4,022,109	2006	1979	.422	63.4
4	4,087,458	2022	1999	.417	63.3
5	4,177,081	2044	2023	.412	63.1
6	4,288,988	2071	2053	.406	62.9
7	4,438,598	2107	2095	.398	62.4

B1	B17	B18	B19	B20	B21
x	T'_{o6} $\frac{1960}{\text{Col. B5}}$	$\Delta T'$ 1960 minus Col.B17	ΔT Col. B18 x Col. B16	L_T Btu/lb.gas Col. B19 x Col. 77	L_T Btu/lb.air Col. B20 x Col. 61
0	1702	258	164.2	47.2	48.0
1	1704	256	162.7	47.7	49.6
2	1707	253	160.8	48.3	51.4
3	1710	250	158.7	49.1	54.0
4	1713	247	156.3	50.0	57.2
5	1717	243	153.5	51.2	61.7
6	1722	238	149.8	52.6	68.2
7	1728	232	144.8	54.1	78.1

B1 x	B22 2 x Col.B21	B23 b = 31.87 minus Col. B22	B24 c = Col.B21 minus 31.87	B25 b^2 = Col.B23 x Col. B23	B26 4ac = 4 x Col. B21 x Col. B24
0	96.0	-64.1	16.1	4110	3095
1	99.2	-67.3	17.7	4530	3515
2	102.8	-70.9	19.5	5025	4010
3	108.0	-76.1	22.1	5790	4775
4	114.4	-82.5	25.3	6810	5790
5	123.4	-91.5	29.8	8370	7360
6	136.4	-104.5	36.3	10920	9900
7	156.2	-124.3	46.2	15480	14430

B1 x	B27 $b^2 - 4ac$= Col.B25 minus Col.B26	B28 $\sqrt{b^2 - 4ac}$ Square root Col.B27	B29 minus Col. B23 minus Col.B28	B30 W_b $\frac{\text{lbs.air}}{\text{lbs.comp.air}}$ $\frac{\text{Col. B29}}{\text{Col. B22}}$	B31 n .167 x Col. B4 x Col. B30
0	1015	31.9	32.2	.336	3.03
1	1015	31.9	35.4	.357	2.91
2	1015	31.9	39.0	.380	2.76
3	1015	31.9	44.2	.419	2.61
4	1020	31.95	50.6	.442	2.43
5	1010	31.85	59.6	.483	2.23
6	1020	31.95	72.6	.531	1.99
7	1050	31.85	92.4	.591	1.70

Appendix C

Table of calculations assuming a compressor pressure ratio of 2.20 and a burner temperature of 1860°R at point 1 on Figure 1. 19660 RPM = 963 ft./sec.

Temperature rise factor = .35

T_{o2} = 530 + .35(530) = 715.5 R

Using Hecks Gas Tables

18237.7 - 1608.3 x = W_a(295.2)

Columns C1 through C10 show the calculations for the weight of air per pound of fuel and the weight of the total combustion products per pound of fuel.

Columns C11 through C16 show the calculations for R_g.

Columns C17 through C26 show the calculations for C_p and k_g using information obtained from the graphs of C_p for the products of combustion and the formula.

$$C_p = \frac{k}{k-1}\ \frac{R}{J}$$

$$\text{or } k = \frac{\frac{J}{R} C_p}{\frac{J}{R} C_p - 1}$$

Columns C27 through C33 show the calculations for nozzle throat velocity and turbine efficiency.

Columns C34 through C41 show the calculations for turbine work.

In the same manner as in Appendix I, an overall heat balance gives:

$$(\text{Col. C41})(W_b)^2 + \left[44.60 - 2(\text{Col. C41})\right] W_b + \text{Col. C41} - 44.60 = 0$$

let a = Col. C41 let b = 44.60 - 2(Col. C41)

let c = Col. C41-44.60

and solve for W_b

Columns C42 through C50 show the calculations for W_b in pounds of air per pound of compressor air.

In the same manner as in Appendix I

$$n = W_a \times W_b \times .2335$$

Columns 51 and 52 show the calculation for thermal efficiency.

C1	C2	C3	C4	C5	C6
x	Δh H_2O	18237.7 minus Col. C2	W_a Col. C3 ÷ 295.2	Total O_2 Col. C4 x .23	Free O_2 Col. C5 minus 3.466
0	0	18237.7	61.78	14.21	10.74
1	1608.3	16629.4	56.33	12.96	9.494
2	3216.6	15021.1	50.88	11.70	8.234
3	4824.9	13412.8	45.44	10.45	6.984
4	6433.2	11804.5	39.99	9.198	5.732
5	8041.5	10196.2	34.45	7.924	4.458
6	9649.8	8587.9	29.09	6.691	3.225
7	11258.1	6979.6	23.64	5.437	1.971
8	12866.4	5371.3	18.20	4.186	0.720
9	14474.7	3763.0	12.75	2.932	--

C1	C7	C8	C9	C10	C11
x	N_2 Col.C4 x .77	CO_2	H_2O Col. C1 + 1.35	Total comb. prod. Col.C6 + Col. C7 + Col.C8+Col.C9	O_2 Col.C6 x 48.31
0	47.57	3.116	1.35	62.78	518.85
1	43.37	3.116	2.35	58.33	458.66
2	39.18	3.116	3.35	53.88	397.78
3	34.99	3.116	4.35	49.44	337.40
4	30.79	3.116	5.35	44.99	276.91
5	26.53	3.116	6.35	40.45	215.37
6	22.40	3.116	7.35	36.09	155.80
7	18.20	3.116	8.35	31.64	95.22
8	14.01	3.116	9.35	27.20	34.78

C1	C12	C13	C14	C15	C16
x	N_2 Col. C7 x 55.16	CO_2 Col. C8 x 35.13	H_2O Col. C9 x 85.81	Total Col. C11+Col. C12 + Col. C13 + Col. C14	R_g Col. C15 ÷ Col. C10
0	2623.96	109.47	115.84	3368.12	53.65
1	2392.29	109.47	201.65	3162.07	54.21
2	2161.17	109.47	287.46	2955.88	54.86
3	1930.05	109.47	373.27	2743.19	55.49
4	1698.38	109.47	459.08	2543.84	56.54
5	1463.39	109.47	544.89	2333.12	57.68
6	1235.58	109.47	630.70	2131.55	59.06
7	1003.91	109.47	716.51	1925.11	60.84
8	772.79	109.47	802.32	1719.36	63.21

C1	C17	C18	C19	C20	C21	C22
x	O_2 Col. C6 x .2625	N_2 Col. C7 x .2800	CO_2 Col. C8 x .2973	H_2O Col. C9 x .5533	Total Col. C17+ Col. C18+ Col. C19+ Col. C20	C_p Col. C21 ÷ Col. C10
0	2.819	13.32	.9264	.7470	17.812	.2837
1	2.492	12.14	.9264	1.3003	16.859	.2890
2	2.161	10.97	.9264	1.8536	15.911	.2953
3	1.833	9.797	.9264	2.4069	14.963	.3026
4	1.505	8.621	.9264	2.9602	14.013	.3115
5	1.170	7.428	.9264	3.5135	13.038	.3223
6	.8466	6.272	.9264	4.0668	12.112	.3356
7	.5174	5.096	.9264	4.6201	11.160	.3527
8	.1890	3.923	.9264	5.1734	10.212	.3754

VC

C1	C23	C24	C25	C26	C27
x	$\frac{J}{R}$ $\frac{778}{\text{Col. C16}}$	$\frac{J}{R}C_p$ Col. C23 x Col. C22	$\frac{J}{R}C_{p-1}$ Col. C24 minus 1	k_g $\frac{\text{Col. C24}}{\text{Col. C25}}$	$1+\frac{k-1}{2}$
0	14.50	4.114	3.114	1.321	1.1605
1	14.35	4.147	3.147	1.318	1.159
2	14.18	4.187	3.187	1.314	1.157
3	14.02	4.242	3.242	1.308	1.154
4	13.76	4.286	3.286	1.304	1.152
5	13.49	4.348	3.348	1.299	1.1495
6	13.17	4.419	3.419	1.292	1.146
7	12.79	4.511	3.511	1.285	1.1425
8	12.31	4.621	3.621	1.276	1.138

C1	C28	C29	C30	C31	C32
x	$k_g R_g$ Col. C26 x Col. C16	$k_g R_g$x32.17 x 1860 = 59836 x Col. C28	$\frac{\text{Col. C29}}{\text{Col. C27}}$	v_n ft/sec. $\sqrt{\text{Col. C30}}$	$\frac{W}{v_n}$ $\frac{963}{\text{Col. C31}}$
0	70.872	4,240,697	3,654,198	1912	.504
1	71.449	4,275,224	3,688,718	1920	.502
2	72.086	4,313,338	3,728,036	1931	.499
3	72.581	4,342,957	3,763,394	1940	.496
4	73.728	4,411,589	3,829,504	1957	.4925
5	74.926	4,483,272	3,900,193	1975	.4875
6	76.306	4,565,846	3,984,159	1996	.483
7	78.179	4,677,919	4,094,459	2023	.476
8	80.656	4,826,132	4,240,889	2059	.468

C1	C33	C34	C35	C36	C37
x	n_T from Fig.2	$\frac{k-1}{k}$ $\frac{\text{Col.C26-1}}{\text{Col. C26}}$	$2.20^{\frac{k-1}{k}}$	$T_{o_6'}$ $\frac{1860}{\text{Col.C35}}$	$\Delta T'$ 1860 minus Col.C36
0	62.7	.243	1.211	1537	323
1	62.8	.2425	1.210	1539	321
2	62.9	.239	1.208	1541	319
3	63.1	.236	1.204	1544	316
4	63.2	.233	1.202	1548	312
5	63.4	.230	1.199	1552	308
6	63.6	.226	1.195	1557	303
7	63.8	.222	1.191	1562	298
8	64.0	.217	1.187	1567	293

C1	C38	C39	C40	C41	C42
x	ΔT Col.C37 x Col.C33	L_T Btu/lb.gas Col.C38x Col.C22	Ratio $\frac{\text{Comb.prod.}}{\text{inlet air}}$ $\frac{\text{Col. C10}}{\text{Col. C4}}$	L_T Btu/lb. inlet air Col. C40 x Col. C39	2 x Col. C41
0	202.5	57.3	1.016	58.2	116.4
1	201.5	58.0	1.035	60.0	120.0
2	200.5	59.2	1.058	62.6	125.2
3	199	60.2	1.088	65.5	131.0
4	197	61.3	1.125	69.0	138.0
5	195	62.9	1.173	73.8	147.6
6	193	64.7	1.240	80.1	160.2
7	190	67.0	1.340	89.8	179.6
8	187.5	70.4	1.494	105.0	210.0

C1 x	C43 b = 44.60 minus Col.C42	C44 c = Col. C41 minus 44.60	C45 b^2= Col.C43 x Col.C43	C46 4ac = 4 x Col. C41 x Col.C44	C47 b^2-4ac = Col.C45 minus Col. C46
0	-71.8	13.60	5150	3170	1980
1	-75.4	15.40	5680	3700	1980
2	-80.6	18.00	6500	4510	1990
3	-86.4	20.90	7460	5480	1980
4	-93.4	24.40	8720	6740	1980
5	-103.0	29.20	10610	8620	1990
6	-115.6	35.50	13330	11380	1950
7	-135.0	45.20	18230	16230	2000
8	-165.4	60.40	27400	25400	2000

C1 x	C48 $\sqrt{b^2 - 4ac}$ Square root Col.C47	C49 minus Col.C43 minus Col.C48	C50 W_b lbs.air per lb.comp.air $\frac{\text{Col. C49}}{\text{Col. C42}}$	C51 Col.C50 x Col. C4	C52 n Col. C51 x .2335
0	44.6	27.2	.233	14.42	3.37
1	44.6	30.8	.257	14.48	3.38
2	44.65	35.95	.287	14.60	3.41
3	44.6	41.8	.319	14.50	3.385
4	44.6	48.8	.354	14.16	3.30
5	44.65	58.35	.396	13.65	3.185
6	44.3	71.3	.445	12.95	3.02
7	44.7	90.3	.503	11.90	2.78
8	44.7	120.7	.574	10.44	2.44

Appendix D

Table of calculations assuming a compressor pressure ratio of 2.20 and a burner temperature of 1760°R at point 1 on Figure 1. 19660 RPM = 963 ft/sec.

Temperature rise factor = .35

$T_{o2} = 530 + .35(530) = 715.5°R$

Using Hecks Gas Tables

$18316.5 - 1553.6x = 268.8 W_a$

Columns D1 through D10 show the calculations for the weight of air per pound of fuel and the weight of the total combustion products per pound of fuel.

Columns D11 through D16 show the calculations for R_g.

Columns D17 through D26 show the calculations for C_p and k_g using information obtained from the graphs of C_p for the products of combustion and the formula

$$C_p = \frac{k}{k-1} \quad \frac{R}{J}$$

$$\text{or } k = \frac{\frac{J}{R} C_p}{\frac{J}{R} C_p - 1}$$

Columns D27 through D33 show the calculations for nozzle throat velocity and turbine efficiency.

Columns D34 through D41 show the calculations for turbine work.

In the same manner as in Appendix I an overall heat balance gives:

$$(\text{Col.}D_{41})(W_b)^2 + \left[44.60 - 2(\text{Col.}D41)\right] W_b + \text{Col.}D41 - 44.60 = 0$$

let a = Col. D41 let b = 44.60 - 2(Col.D41)

let c = Col. D41-44.60

and solve for W_b

Columns D42 through D50 show the calculation for W_b in pounds of air per pound of compressor air.

In the same manner as in appendix 1

$$n = W_a \times W_b \times .2335$$

Columns D51 and D52 show the calculation for thermal efficiency.

D1	D2	D3	D4	D5	D6
x	Δh H_2O	18316.54 minus Col. D2	N_a Col. D3 ÷ 268.8	Total O_2 Col. D4 x .23	Free O_2 Col. D5 minus 3.466
0	0	18316.54	68.14	15.672	12.206
1	1553.6	16762.94	62.36	14.343	10.877
2	3107.2	15209.34	56.58	13.013	9.547
3	4660.8	13655.74	50.80	11.684	8.218
4	6214.4	12102.14	45.02	10.355	6.889
5	7768.0	10548.54	39.24	9.025	5.559
6	9321.6	8995.14	33.46	7.696	4.230
7	10875.2	7441.54	27.68	6.366	2.900
8	12428.8	5887.94	21.90	5.037	1.571
9	13982.4	4334.34	16.12	3.708	0.242
10	15536.0	2780.74	10.35	2.380	--

D1	D7	D8	D9	D10	D11
x	N_2 Col.D4 x .77	CO_2	H_2O Col. D1 + 1.35	Total comb.prod. Col.D6+Col.D7+ Col.D8+Col.D9	O_2 Col.D6 x 48.31
0	52.468	3.116	1.35	69.14	598.67
1	48.017	3.116	2.35	64.36	525.47
2	43.567	3.116	3.35	59.58	461.22
3	39.116	3.116	4.35	54.80	397.01
4	34.665	3.116	5.35	50.02	322.81
5	30.215	3.116	6.35	45.24	268.56
6	25.764	3.116	7.35	40.46	204.35
7	21.314	3.116	8.35	35.68	140.10
8	16.863	3.116	9.35	30.90	75.90
9	12.412	3.116	10.35	26.12	11.69

D1 x	D12 N_2 Col. D7 x 55.16	D13 CO_2 Col. D8 x 35.13	D14 H_2O Col. D9 x 85.81	D15 Total Col.D11 + Col.D12 + Col.D13+Col.D14	D16 R_g $\frac{\text{Col.D15}}{\text{Col.D10}}$
0	2894.13	109.47	115.84	3709.11	53.65
1	2648.62	109.47	201.65	3485.21	54.15
2	2403.16	109.47	287.46	3261.31	54.74
3	2157.64	109.47	373.27	3037.39	55.43
4	1912.12	109.47	459.08	2813.48	56.25
5	1666.66	109.47	544.89	2589.58	57.24
6	1421.14	109.47	630.70	2365.66	58.45
7	1175.68	109.47	716.51	2141.76	60.03
8	930.16	109.47	802.32	1917.85	62.07
9	684.65	109.47	888.13	1693.94	64.85

D1 x	D17 O_2 Col. D6 x .2584	D18 N_2 Col. D7 x .2775	D19 CO_2 Col. D8 x .2936	D20 H_2O Col. D9 x .5428	D21 Total Col.D17 + Col.D18 + Col.D19+Col.D20
0	3.1540	14.5599	.9149	.7328	19.3616
1	2.8106	13.3247	.9149	1.2756	18.3267
2	2.4669	12.0898	.9149	1.8184	17.2900
3	2.1235	10.8547	.9149	2.3612	16.2543
4	1.7801	9.6195	.9149	2.9040	15.2185
5	1.4364	8.3447	.9149	3.4468	14.1428
6	1.0930	7.1495	.9149	3.9896	13.1470
7	.7494	5.9146	.9149	4.5324	12.1113
8	.4059	4.6795	.9149	5.0752	11.0755
9	.0625	3.4443	.9149	5.6180	10.0397

D1 x	D22 C_p Col.D21 / Col.D10	D23 $\frac{J}{R}$ 778 / Col. D16	D24 $\frac{J}{R} C_p$ Col. D23 x Col. D22	D25 $\frac{J}{R} C_p - 1$ Col. D24 minus 1	D26 k_g Col. D24 / Col. D25
0	.2800	14.50	4.060	3.060	1.327
1	.2848	14.37	4.093	3.093	1.323
2	.2902	14.21	4.124	3.124	1.320
3	.2966	14.04	4.164	3.164	1.316
4	.3042	13.83	4.207	3.207	1.312
5	.3126	13.59	4.248	3.248	1.308
6	.3249	13.31	4.324	3.324	1.301
7	.3394	12.96	4.399	3.399	1.294
8	.3584	12.53	4.491	3.491	1.286
9	.3845	12.00	4.614	3.614	1.276

D1 x	D27 $1 + \frac{k-1}{2}$	D28 $k_g R_g$ Col.D26 x Col.D16	D29 kgRgx 32.17 x 1760 = 56,619 x Col. D28	D30 Col.D29 / Col.D27	D31 v_n ft/sec. $\sqrt{\text{Col.D30}}$
0	1.1635	71.194	4,030,933	3,464,489	1861
1	1.1615	71.640	4,056,185	3,492,194	1869
2	1.160	72.257	4,091,119	3,526,827	1878
3	1.158	72.946	4,130,130	3,566,606	1889
4	1.156	73.800	4,178,482	3,614,604	1901
5	1.154	74.870	4,239,065	3,673,366	1917
6	1.1505	76.043	4,305,479	3,742,268	1934
7	1.147	77.679	4,398,107	3,834,444	1958
8	1.143	79.822	4,519,442	3,954,017	1988
9	1.138	82.749	4,685,165	4,117,017	2029

D1	D32	D33	D34	D35	D36
x	$\frac{W}{V_n}$ $\frac{963}{\text{Col.D31}}$	n_T from Fig.2	$\frac{k-1}{k}$ $\frac{\text{Col.D26}-1}{\text{Col.D26}}$	$2.20^{\frac{k-1}{k}}$	$T_{o'6}$ $\frac{1760}{\text{Col.D35}}$
0	.517	61.8	.2465	1.215	1450
1	.515	61.9	.244	1.212	1452
2	.513	62.1	.242	1.210	1454
3	.510	62.3	.240	1.208	1457
4	.507	62.5	.238	1.206	1460
5	.502	62.8	.2355	1.204	1463
6	.498	63.0	.2315	1.200	1468
7	.492	63.2	.227	1.196	1472
8	.484	63.5	.222	1.191	1478
9	.475	63.8	.2165	1.186	1485

D1	D37	D38	D39	D40	D41
x	1760 minus Col.D36	ΔT Col.D37 x Col.D33	L_T Btu/lb. gas Col. D38 x Col.D22	Ratio $\frac{\text{Comb.prod.}}{\text{inlet air}}$ $\frac{\text{Col. D10}}{\text{Col. D4}}$	L_T Btu/lb. inlet air Col. D40 x Col. D39
0	310	191.5	53.6	1.014	54.4
1	308	190.8	54.4	1.032	56.1
2	306	190.0	55.2	1.053	58.1
3	303	189.0	56.1	1.079	60.5
4	300	187.5	57.0	1.111	63.4
5	297	186.0	58.1	1.153	67.0
6	292	184.0	59.8	1.210	72.4
7	288	182.0	61.8	1.29	79.7
6	282	179.0	64.2	1.41	90.5
9	275	175.5	67.5	1.62	109.3

D1	D42	D43	D44	D45	D46	D47
x	2 x Col.D41	b = 44.60 - Col.D42	c = Col.D41 -44.60	b^2= Col.D43 x Col.D43	4ac=4 x Col.D41x Col.D44	b^2- 4ac = Col. D45 - Col.D46
0	108.8	-64.2	9.8	4122	2130	1992
1	112.2	-67.6	11.5	4570	2585	1985
2	116.2	-71.6	13.5	5127	3143	1984
3	121.0	-76.4	15.9	5837	3845	1992
4	126.8	-82.2	18.8	6757	4765	1992
5	134.0	-89.4	22.4	7992	6005	1987
6	144.8	-100.2	27.8	10040	8050	1990
7	159.4	-114.8	35.1	13179	11200	1979
8	181.0	-136.4	45.9	18605	16600	2005
9	218.6	-174.0	64.7	30276	28300	1976

D1	D48	D49	D50	D51	D52
x	$\sqrt{b^2 - 4ac}$ Square root Col.D47	minus Col.D43 minus Col.D48	W_b lbs.air per lb. comp. air $\frac{\text{Col. D49}}{\text{Col. D42}}$	Col. D50 x Col. D4	Eff. = Col. D51 x .2335
0	44.6	19.6	.180	12.28	2.86
1	44.6	23.0	.205	12.78	2.98
2	44.6	27.0	.232	13.15	3.07
3	44.6	31.8	.263	13.38	3.12
4	44.6	37.6	.297	13.38	3.12
5	44.6	44.8	.334	13.02	3.04
6	44.6	55.6	.384	12.87	3.00
7	44.5	70.3	.441	12.21	2.87
8	44.8	91.6	.506	11.09	2.585
9	44.5	129.5	.592	9.56	2.233

Appendix 2

Tables of calculations assuming a compressor pressure ratio of 2.20 and a burner temperature of 1660°R at point 1 on Figure 1. 19660 RPM = 963 ft/sec.

Temperature rise factor = .35

$T_{02} = 530 + .35(530) = 715.5^{\circ}R.$

Using Hecks Gas Tables

$18394.2 - 1499.7x = 240.8\ W_a$

Columns E1 through E10 show the calculations for the weight of air per pound of fuel and the weight of the total combustion products per pound of fuel.

Columns E11 through E16 show the calculations for R_g.

Columns E17 through E26 show the calculations for C_p and k_g using information obtained from the graphs of C_p for the products of combustion and the formula

$$C_p = \frac{k}{k-1} \frac{R}{J}$$

$$\text{or } k = \frac{\frac{J}{R} C_p}{\frac{J}{R} C_{p-1}}$$

Columns E27 through E33 show the calculations for nozzle throat velocity and turbine efficiency.

Columns E34 through E41 show the calculations for turbine work.

In the same manner as in Appendix 1 an overall heat balance gives:

$$(\text{Col. E41})(W_b)^2 + \left[44.60 - 2(\text{Col. E41})\right] W_b + \text{Col. E41} - 44.60 = 0$$

let a = Col. E41 let b = 44.60 - 2(Col. E41)

let c = Col. E41 - 44.60

and solve for W_b

Columns E42 through E50 show the calculations for W_b in pounds of air per pound of compressor air.

In the same manner as in Appendix 1

$$n = W_a \times W_b \times .2335$$

Columns D51 and D52 show the calculations for thermal efficiency.

E1	E2	E3	E4	E5
x	Δh H_2O	18394.2 minus Col. E2	W_a Col. E3 ÷ 240.8	Total O_2 Col. E4 x .23
0	0	18394.2	76.39	17.570
1	1499.7	16894.5	70.16	16.137
2	2999.4	15394.8	63.93	14.704
3	4499.1	13895.1	57.70	13.271
4	5998.8	12395.4	51.48	11.840
5	7498.5	10895.7	45.25	10.408
6	8998.2	9396.0	39.02	8,975
7	10497.9	7896.3	32.79	7.541
8	11997.6	6396.6	26.56	6.109
9	13497.3	4896.9	20.34	4.679
10	14997.0	3397.2	14.11	3.245
11	16496.7	1897.5	7.88	1.812

E1	E6	E7	E8	E9	E10
x	Free O_2 Col. E5 minus 3.466	N_2 Col. E4 x .77	CO_2	H_2O Col. E1 + 1.35	Total comb. prod. Col. E6+Col. E7+ Col. E8+Col. E9
0	14.104	58.820	3.116	1.35	77.39
1	12.671	54.023	3.116	2.35	72.16
2	11.238	49.226	3.116	3.35	66.93
3	9.805	44.429	3.116	4.35	61.70
4	8.374	39.640	3.116	5.35	56.48
5	6.942	34.842	3.116	6.35	51.25
6	5.509	30.045	3.116	7.35	46.02
7	4.075	25.248	3.116	8.35	40.79
8	2.643	20.451	3.116	9.35	35.56
9	1.213	15.662	3.116	10.35	30.34

E1	E11	E12	E13	E14	E15
x	O_2 Col. E6 x 48.31	N_2 Col. E7 x 55.16	CO_2 Col. E8 x 35.13	H_2O Col. E9 x 85.81	Total Col. E11+Col. E12 +Col. E13+Col. E14
0	681.36	3244.51	109.47	115.84	4151.18
1	612.14	2979.91	109.47	201.65	3903.17
2	542.91	2715.31	109.47	287.46	3655.15
3	473.68	2450.70	109.47	373.27	3407.12
4	404.55	2186.54	109.47	459.08	3159.64
5	335.37	1921.88	109.47	544.89	2911.61
6	266.14	1657.28	109.47	630.70	2663.59
7	196.86	1392.68	109.47	716.51	2415.52
8	127.68	1128.08	109.47	802.32	2167.55
9	58.60	863.92	109.47	888.13	1920.12

E1	E16	E17	E18	E19	E20
x	R_g Col. E15 / Col. E10	O_2 Col. E6 x .2556	N_2 Col. E7 x .2739	CO_2 Col. E8 x .2895	H_2O Col. E9 x .5333
0	53.64	3.6050	16.1108	.9021	.7200
1	54.09	3.2387	14.7969	.9021	1.2533
2	54.61	2.8724	13.4830	.9021	1.7866
3	55.22	2.5062	12.1691	.9021	2.3199
4	55.94	2.1404	10.8574	.9021	2.8532
5	56.81	1.7744	9.5432	.9021	3.3865
6	57.86	1.4081	8.2293	.9021	3.9198
7	59.22	1.0416	6,9154	.9021	4.4531
8	60.95	.6756	5.6015	.9021	4.9864
9	63.29	.3100	4,2898	.9021	5.5197

E1	E21	E22	E23	E24	E25
x	Total Col.E17+Col.E18 +Col.E19+Col.E20	C_p Col.E21 / Col.E10	$\frac{J}{R}$ 778 / Col.E16	$\frac{J}{R} C_p$ Col.E23 x Col. E22	$\frac{J}{R} C_p - 1$ Col. E24 minus 1
0	21.3379	.2757	14.50	3.998	2.998
1	20.1910	.2798	14.38	4.024	3.024
2	19.0441	.2845	14.25	4.067	3.067
3	17.8973	.2901	14.09	4.088	3.088
4	16.7531	.2966	13.91	4.126	3.126
5	15.6062	.3045	13.69	4.169	3.169
6	14.4593	.3142	13.44	4.223	3.223
7	13.3122	.3264	13.14	4.289	3.289
8	12.1656	.3421	12.76	4.365	3.365
9	11.0216	.3633	12.29	4.465	3.465

E1 x	E26 k_g $\frac{\text{Col. E24}}{\text{Col. E25}}$	E27 $1+\frac{k-1}{2}$	E28 $k_g R_g$ Col. E26 x Col. E16	E29 kRx32.17 x 1660 = 53,402 x Col. E28	E30 $\frac{\text{Col. E29}}{\text{Col. E27}}$	E31 v_n ft/sec. $\sqrt{\text{Col. E30}}$
0	1.334	1.167	71.556	3,821,234	3,274,408	1810
1	1.331	1.1655	71.994	3,844,624	3,298,690	1816
2	1.327	1.1635	72.467	3,869,883	3,326,070	1824
3	1.324	1.162	73.111	3,904.274	3,359,960	1833
4	1.320	1.160	73.841	3,943,257	3,399,359	1844
5	1.316	1.158	74.762	3,992,440	3,447,703	1857
6	1.310	1.155	75.823	4,049,100	3,505,714	1872
7	1.304	1.152	77.223	4,123,863	3,579,742	1892
8	1.297	1.1485	79.052	4,221,535	3,675,694	1917
9	1.289	1.1445	81.581	4,356,589	3,806,543	1951

E1 x	E32 $\frac{W}{v_n}$ = $\frac{963}{\text{Col. E31}}$	E33 n_T from Fig. 2	E34 $\frac{k-1}{k}$ $\frac{\text{Col. E26-1}}{\text{Col. E26}}$	E35 $2.20^{\frac{k-1}{k}}$	E36 T_{o6}' $\frac{1660}{\text{Col. E35}}$	E37 1660 minus Col. E36
0	.532	60.7	.250	1.218	1364	296
1	.530	60.8	.249	1.217	1365	295
2	.528	61.0	.2465	1.215	1366	294
3	.525	61.2	.2445	1.213	1368	292
4	.522	61.5	.242	1.210	1371	289
5	.519	61.7	.240	1.208	1374	286
6	.514	62.0	.2365	1.205	1378	282
7	.509	62.3	.233	1.202	1381	279
8	.502	62.8	.229	1.198	1383	275
9	.494	63.2	.224	1.193	1390	270

E1	E38	E39	E40	E41	E42
x	ΔT act Col.E37 x Col.E33	L_T Btu/lb. gas Col.E38 x Col.E22	Ratio $\frac{\text{comb.prod.}}{\text{inlet air}}$ $\frac{\text{Col. E10}}{\text{Col. E4}}$	L_T Btu/lb. inlet air Col. E40 x Col. E 39	2 x Col.E41
0	179.8	49.5	1.011	50.1	100.2
1	179.3	50.2	1.030	51.7	103.4
2	179.3	51.0	1.047	53.4	106.8
3	178.8	51.9	1.070	55.5	111.0
4	178.0	52.8	1.097	57.9	115.8
5	176.5	53.8	1.132	61.0	122.0
6	175.0	54.9	1.180	64.8	129.6
7	173.8	56.7	1.244	70.6	141.2
8	172.8	59.1	1.340	79.2	158.4
9	170.8	62.0	1.490	92.4	184.8

E1	E43	E44	E45	E46	E47
x	b = 44.60 minus Col.E42	c = Col.E41 minus 44.60	b^2 = Col.E43 x Col.E43	4ac = 4 x Col.E41 x Col.E44	b^2 - 4ac = Col. E45 minus Col. 46
0	-55.6	5.5	3095	1102	1993
1	-58.8	7.1	3460	1469	1991
2	-62.2	8.8	3870	1880	1990
3	-66.4	10.9	4410	2420	1990
4	-71.2	13.3	5070	3030	2040
5	-77.4	16.4	5985	4000	1985
6	-85.0	20.2	7220	5235	1985
7	-96.6	26.0	9340	7340	2000
8	-113.8	34.6	12930	10970	1960
9	-140.2	47.8	19650	17680	1970

E1	E48	E49	E50	E51	E52
x	$\sqrt{b^2 - 4ac} = \sqrt{\text{Col. E47}}$	minus Col. E43 minus Col. E48	W_b lbs. air per lb. comp. air $\frac{\text{Col. E49}}{\text{Col. E 42}}$	Col. E50 x Col. E4	Eff. = Col. E51 x .2335
0	44.6	11.0	.1097	8.36	1.95
1	44.6	14.2	.1371	9.61	2.245
2	44.6	17.6	.1650	10.54	2.460
3	44.6	21.8	.1963	11.33	2.645
4	45.1	26.1	.2255	11.61	2.710
5	44.6	32.8	.2685	12.18	2.840
6	44.6	40.4	.3120	12.18	2.840
7	44.7	51.9	.3670	12.03	2.810
8	44.4	69.4	.4380	11.62	2.713
9	44.5	95.7	.5175	10.52	2.455

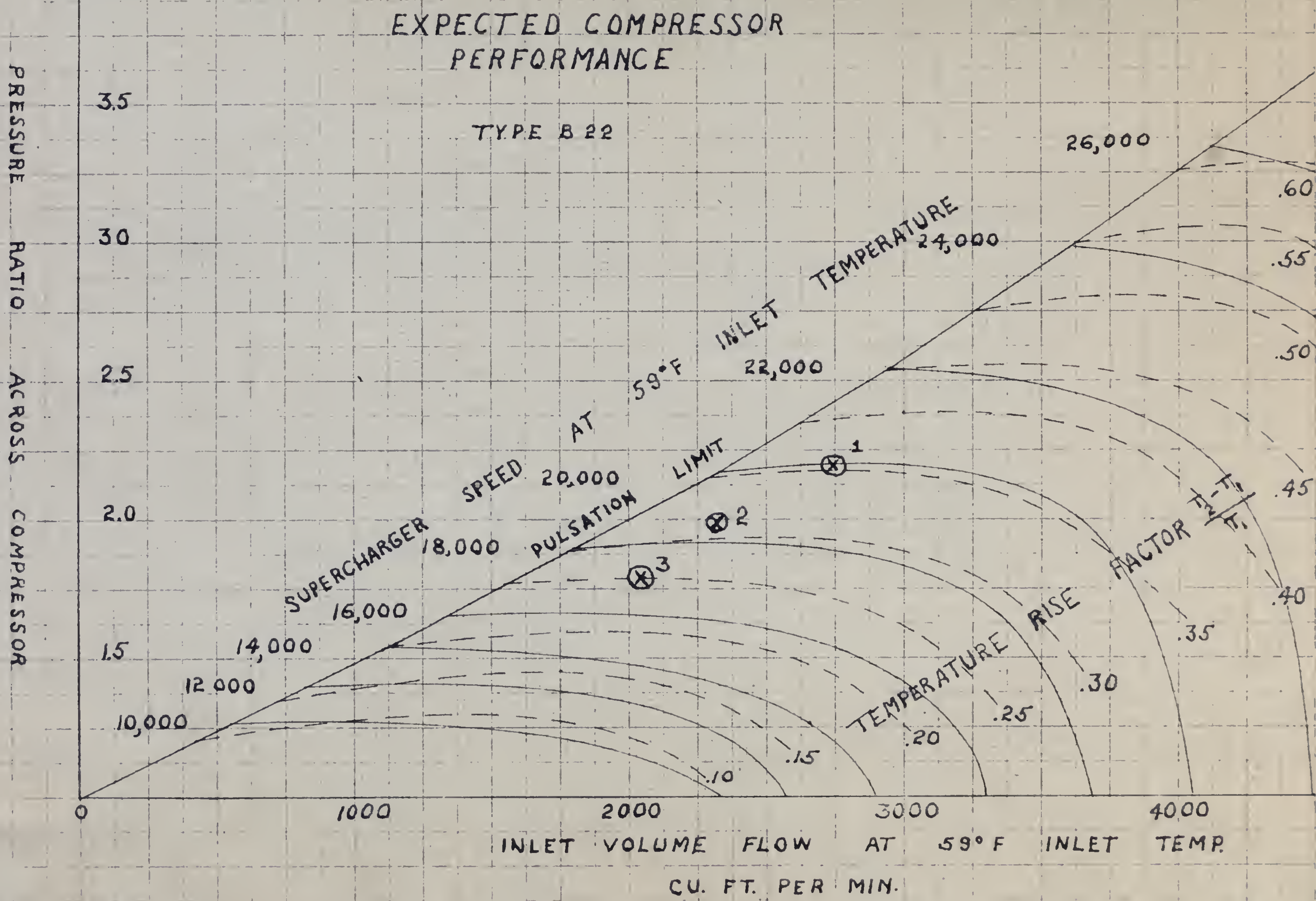


FIG. 1

GENERAL ELECTRIC CO

TYPICAL TURBINE EFFICIENCY
VERSUS PRESSURE RATIO AND VELOCITY RATIO W/V.
(TYPE. B WITH CAST DIAPHRAGM)
EFFICIENCIES BASED ON AVERAGE TEST EFFICIENCIES

FIGURES ON CURVES ARE TURBINE EFFICIENCIES

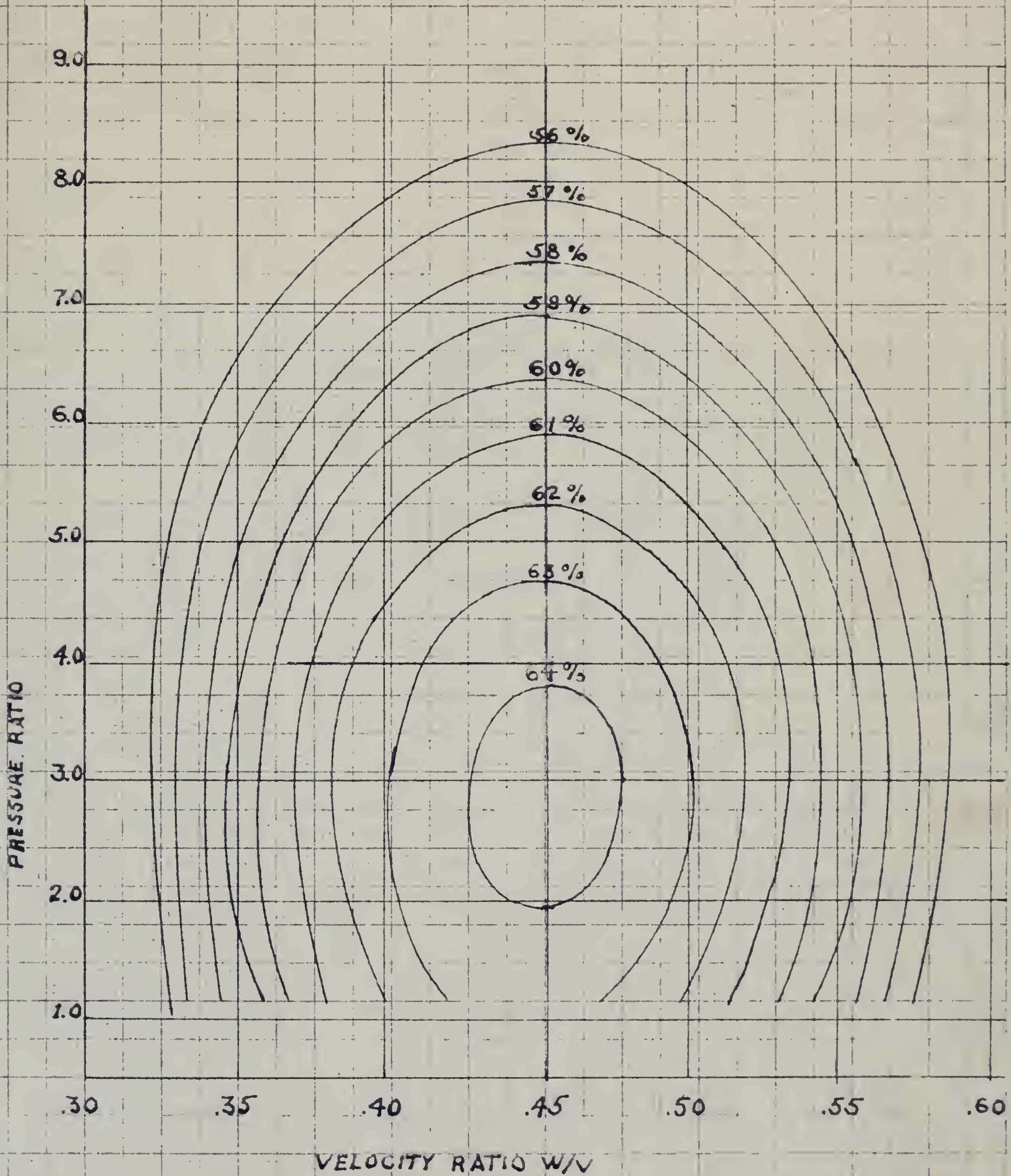


FIG. 2

GENERAL ELECTRIC CO.

EDUCATION GAS TURBINE

TURBINE BLADE SPEED
R.P.M vs. FT. PER SEC.

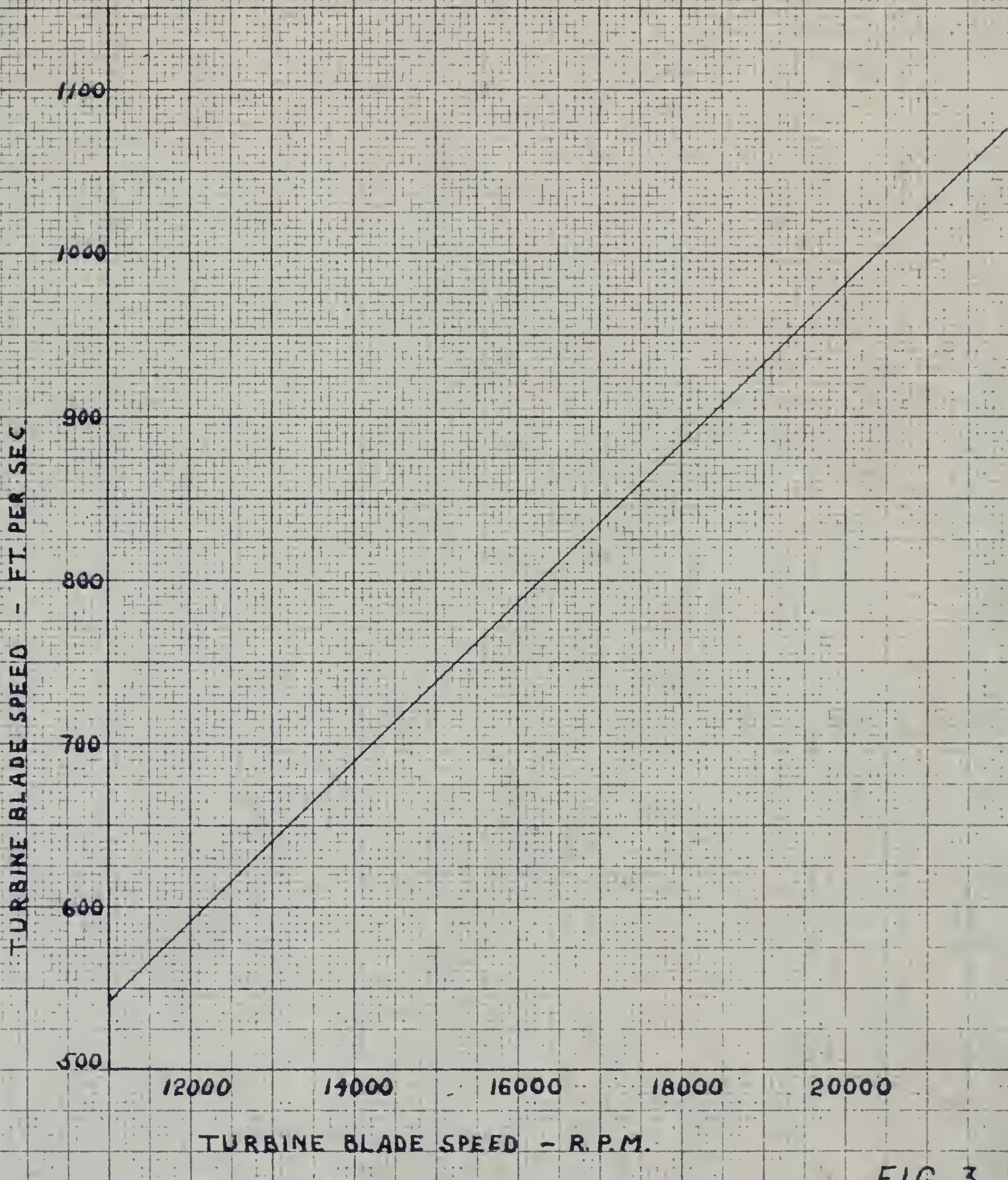


FIG. 3

R. E. TUGEND

EDUCATIONAL GASTURBINE

COMPARISON OF k - RATIO OF SPECIFIC HEATS FOR COMBUSTION GASES AT 1960°R AS DETERMINED FROM HECKS GASTABLES AND FROM GRAPHS OF C_p

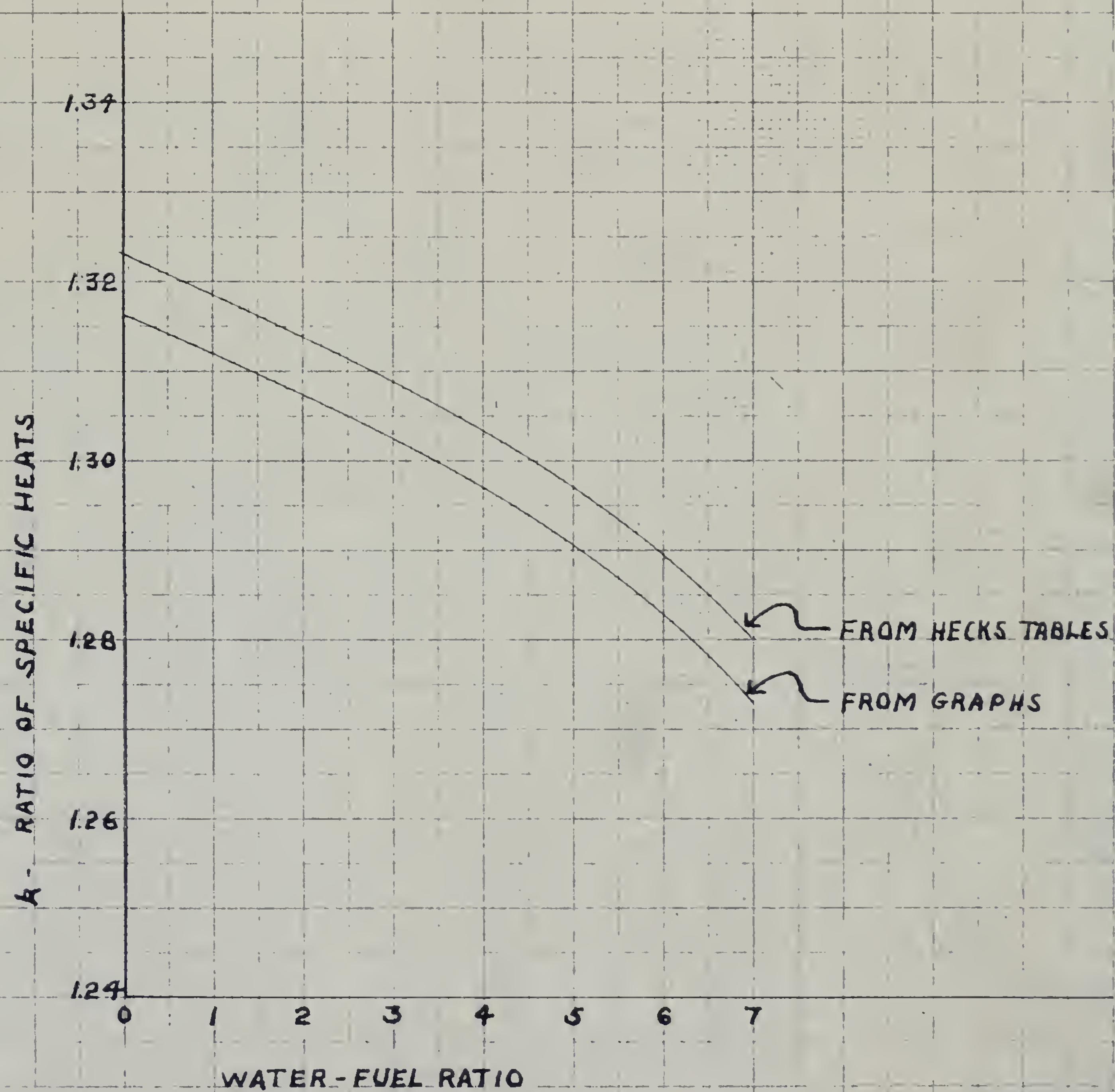


FIG. 4

RETUGEND

EDUCATIONAL GASTURBINE

COMPARISON OF OUTPUT AT 1960 °R
FOR VARIOUS PRESSURE RATIOS

FIGURES ON GRAPH INDICATE PRESSURE RATIOS – $\frac{P_{o5}}{P_{o6}}$

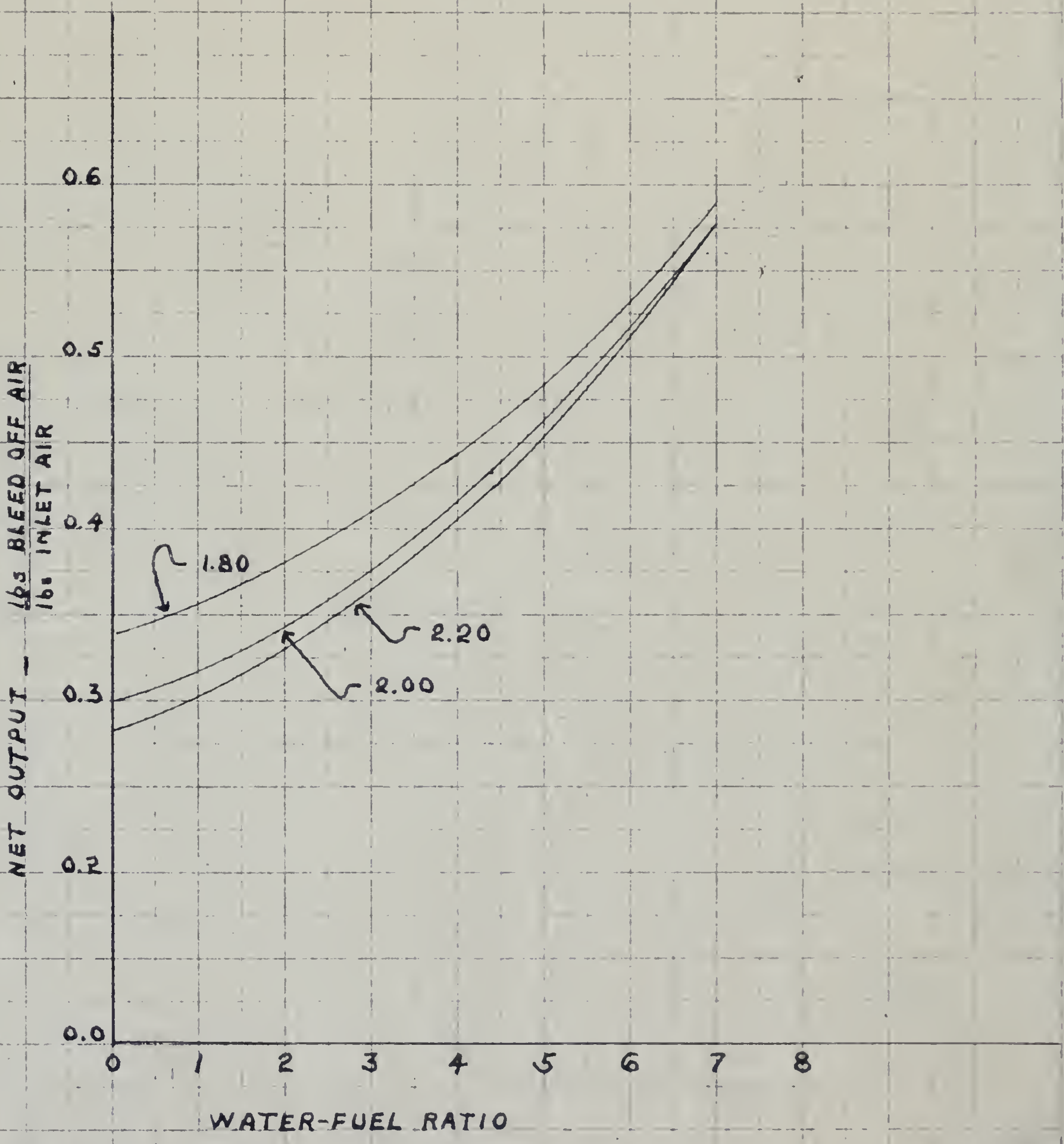


FIG. 5

R. E. TUGEND

EDUCATIONAL GAS TURBINE

RATIO OF SPECIFIC HEATS FOR COMBUSTION GASES AT VARIOUS TEMPERATURES

FIGURES ON GRAPH INDICATE BURNER TEMPERATURES - T_{05}

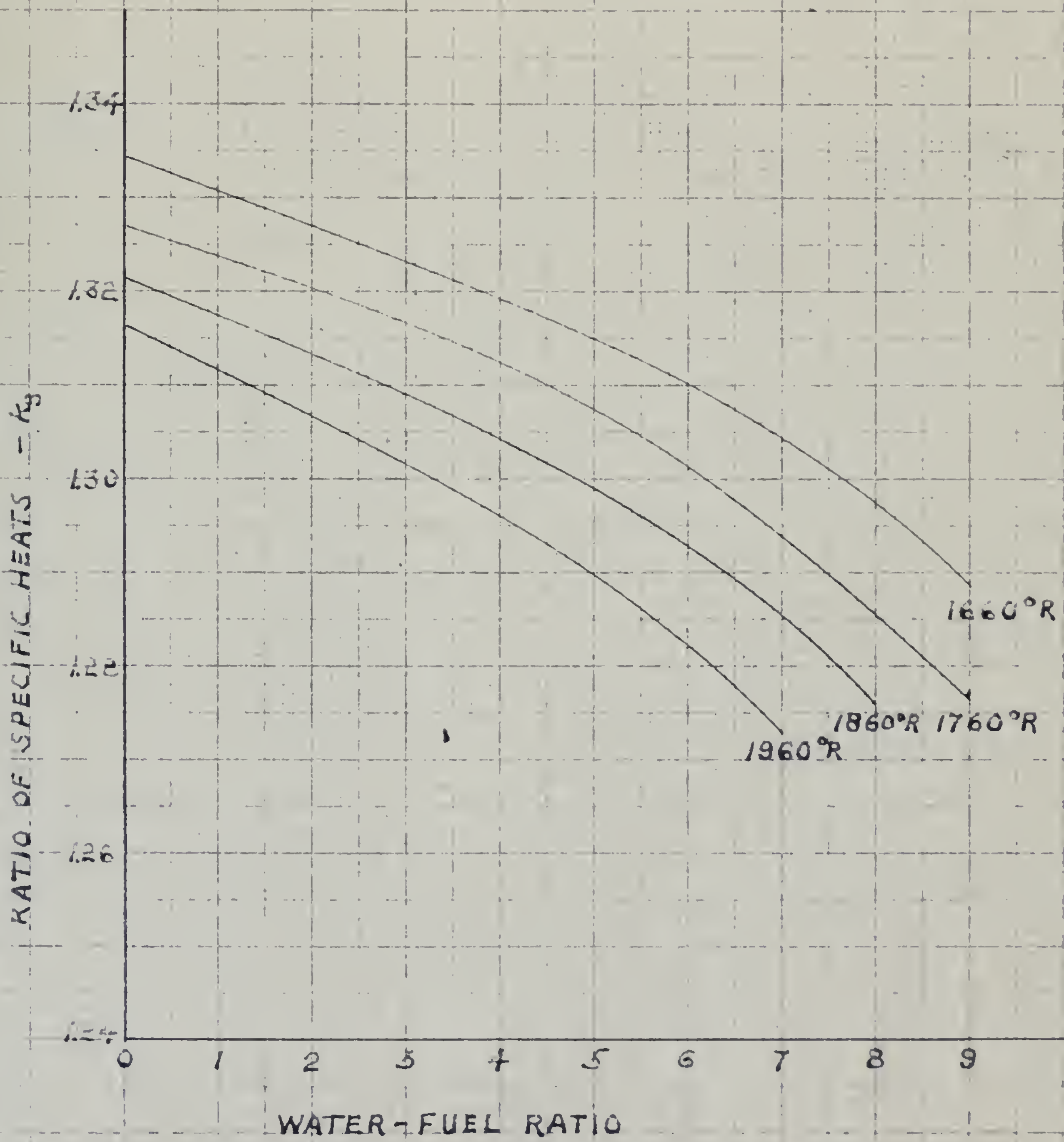


FIG. 6

R.E. TUGEND

EDUCATIONAL GAS TURBINE

COMPARISON OF OUTPUT AT PRESSURE RATIO 2.20 FOR VARIOUS BURNER TEMPERATURES

FIGURES ON GRAPH INDICATE BURNER TEMPERATURE - T_{05}

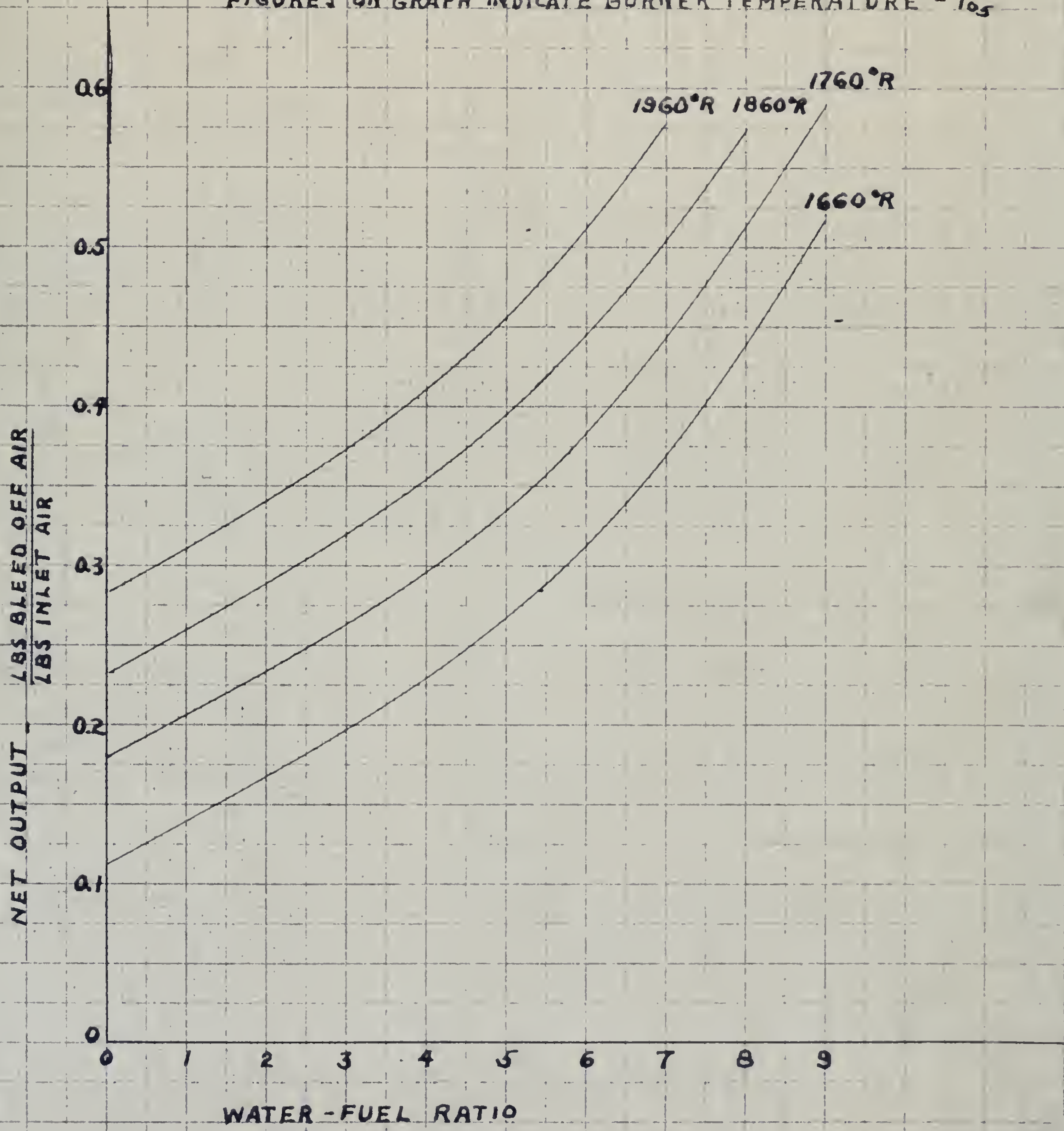


FIG. 7

R.E. TUGEND

EDUCATIONAL GAS TURBINE

COMPARISON OF THERMAL EFFICIENCY AT PRESSURE RATIO 2.20 FOR VARIOUS BURNER TEMPERATURES

FIGURES ON GRAPH INDICATE BURNER TEMPERATURES - T_{o_6}

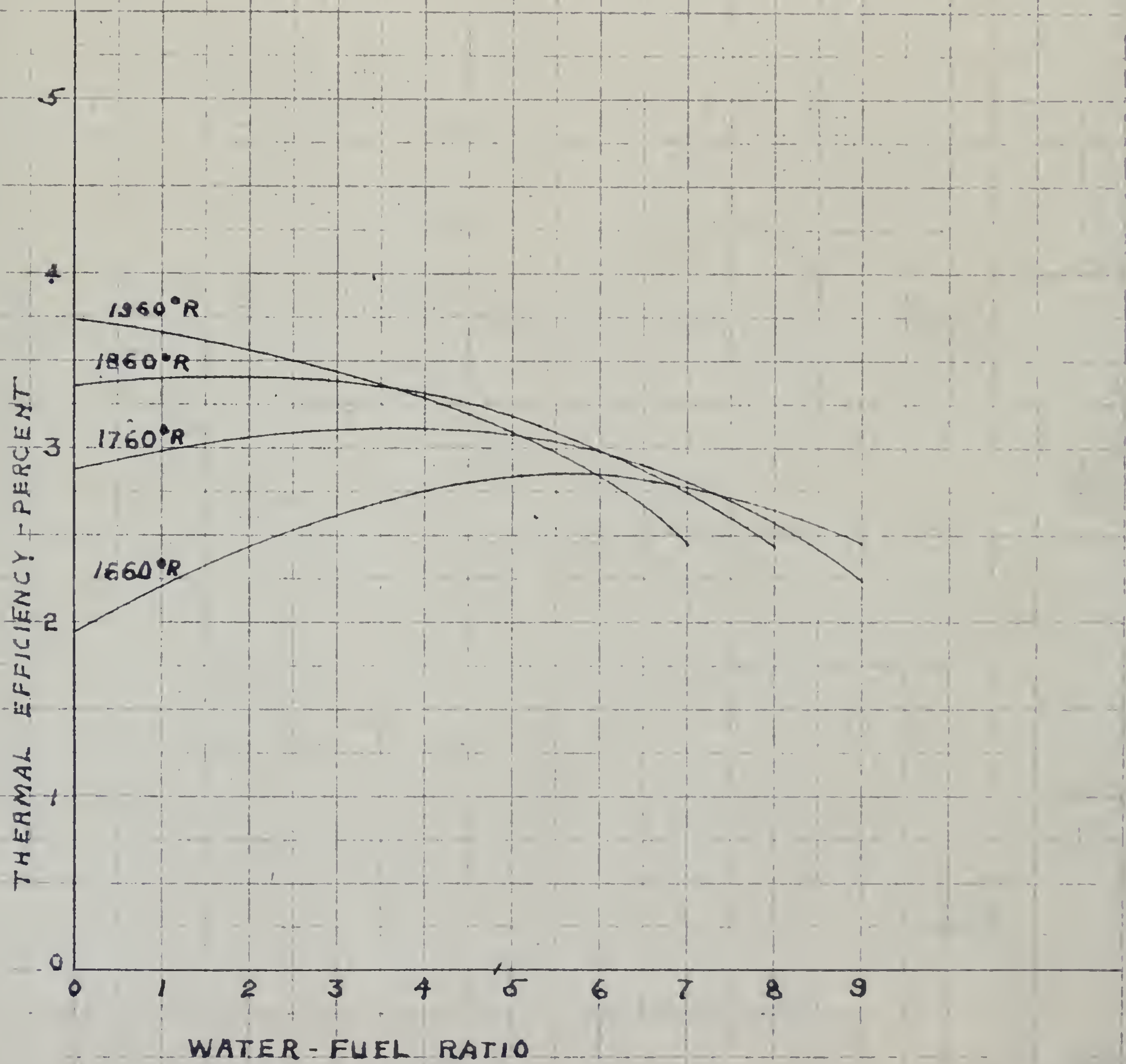


FIG. 8

R.E. TUGEND

EDUCATIONAL GAS TURBINE

COMPARISON OF THERMAL EFFICIENCY AT BURNER TEMPERATURE 1960°R FOR VARIOUS PRESSURE RATIOS

FIGURES ON GRAPH INDICATE PRESSURE RATIO $\frac{P_{05}}{P_{06}}$

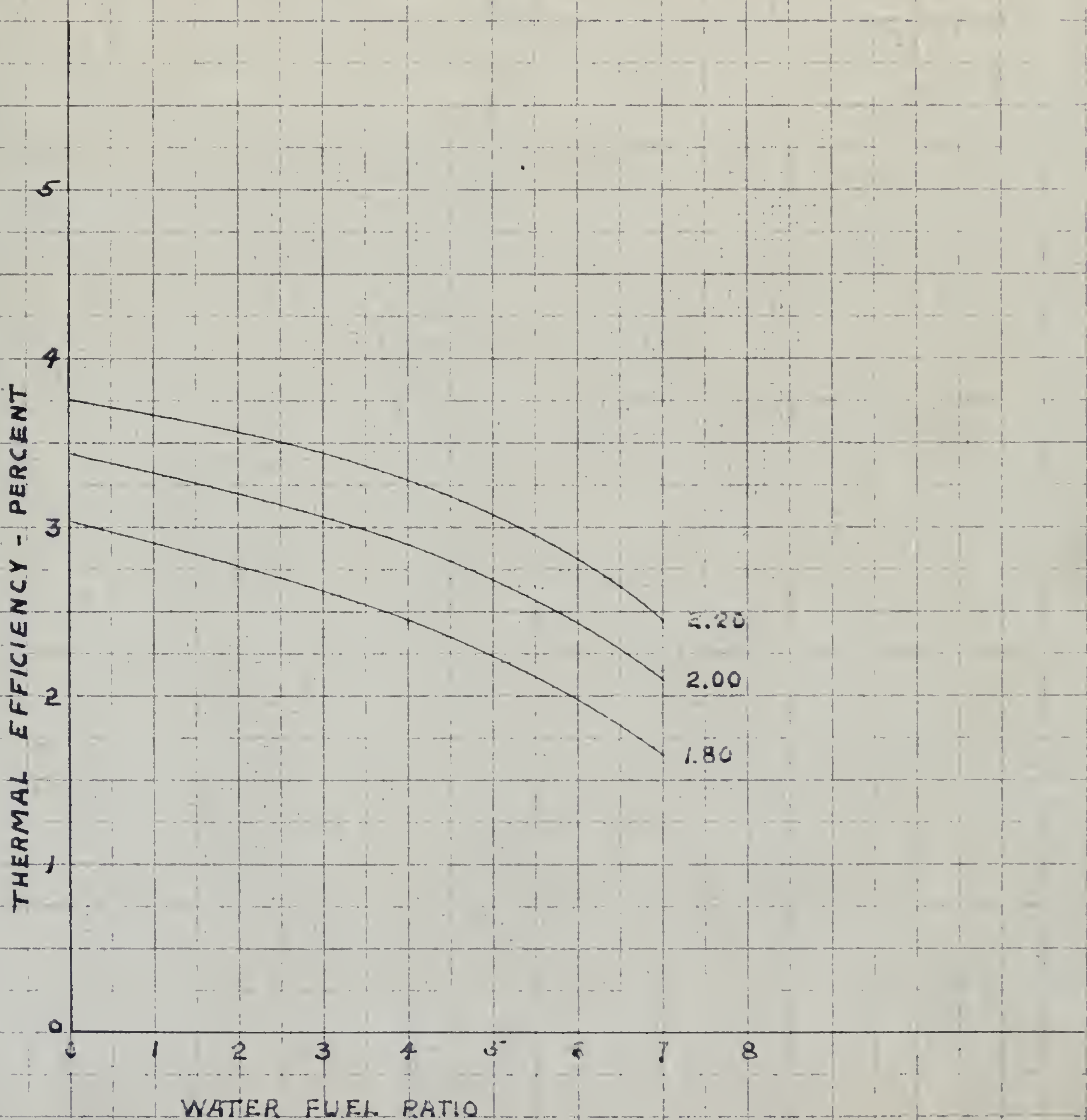
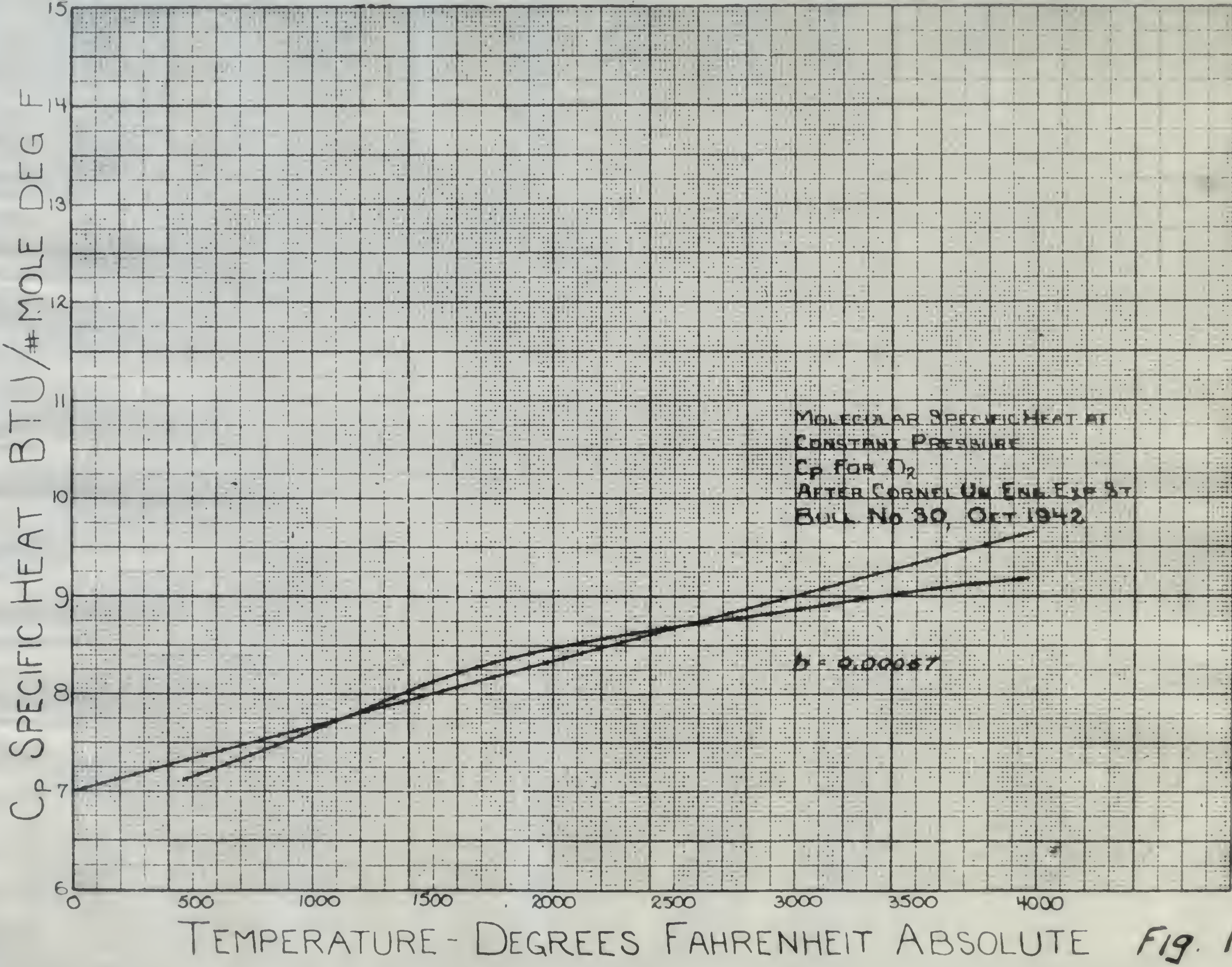


FIG. 9

R.E. TUGEND



MOLECULAR SPECIFIC HEAT AT
CONSTANT PRESSURE
Cp FOR O2
AFTER CORNEL UN ENG EXP ST
BULL NO 30, OCT 1942
b = 0.00067
Cp SPECIFIC HEAT BTU/# MOLE DEG F
TEMPERATURE - DEGREES FAHRENHEIT ABSOLUTE
6
7
8
9
10
11
12
13
14
15
0
500
1000
1500
2000
2500
3000
3500
4000

Fig. 16

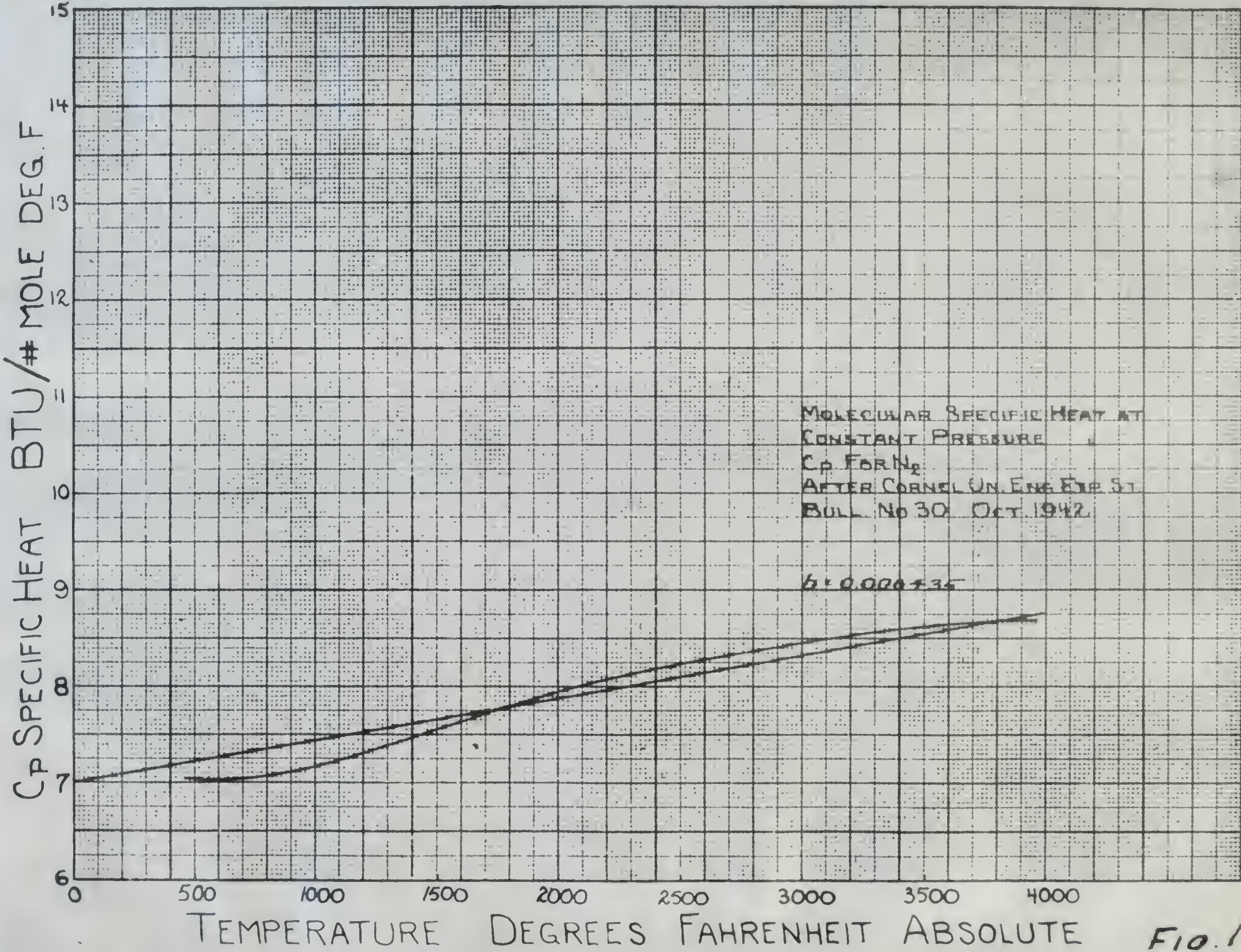

MOLECULAR SPECIFIC HEAT AT
CONSTANT PRESSURE
Cp FOR N2
AFTER CORNEL UN. ENG EXP ST
BULL NO 30 OCT 1942
b = 0.000435
Cp SPECIFIC HEAT BTU/# MOLE DEG. F
TEMPERATURE DEGREES FAHRENHEIT ABSOLUTE
15
14
13
12
11
10
9
8
7
6
0
500
1000
1500
2000
2500
3000
3500
4000

FIG. 17

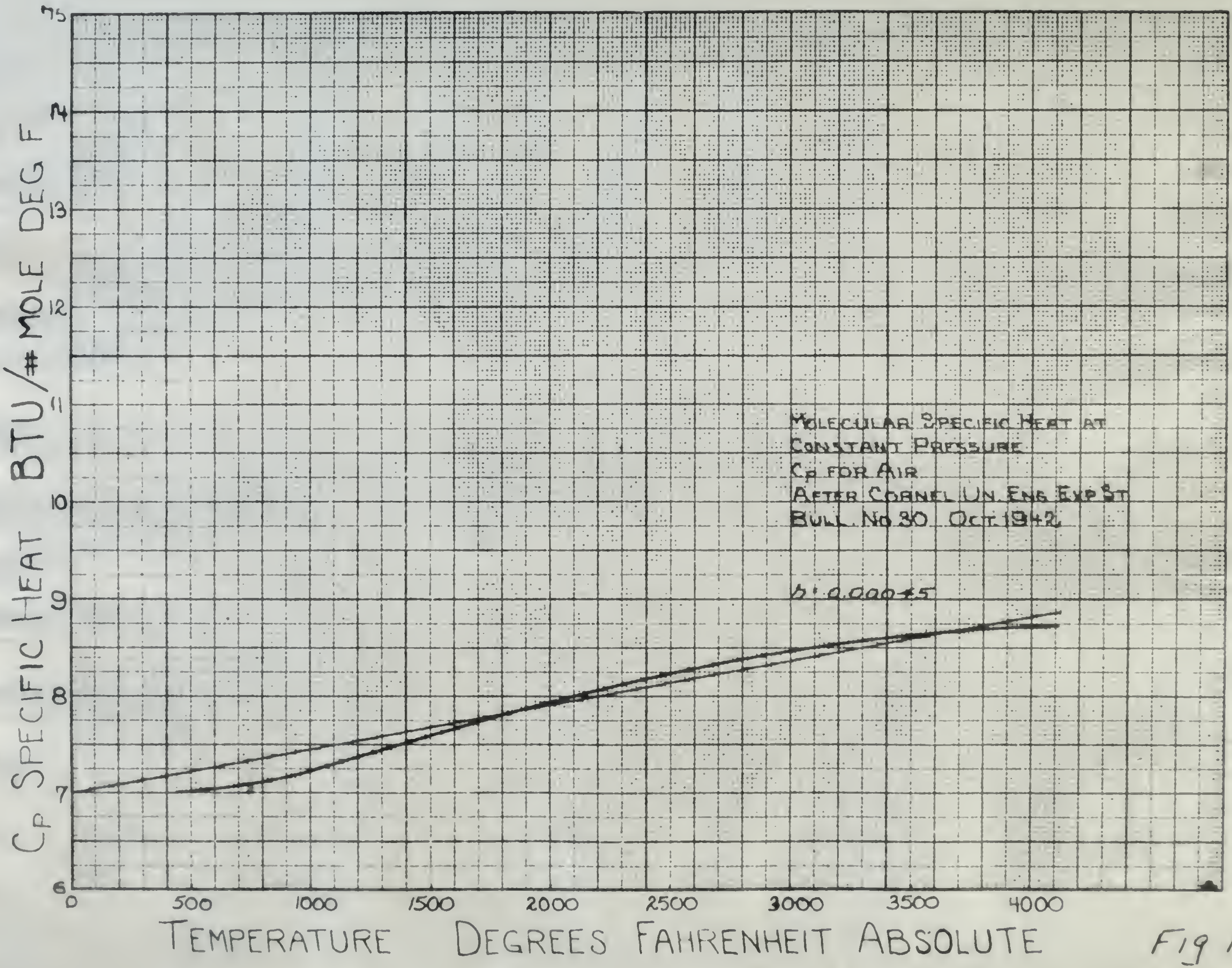

Molecular Specific Heat at
Constant Pressure
Cp for Air
After Cornel Un. Eng Exp St
Bull. No 30 Oct 1942
b: 0.00045
Cp Specific Heat BTU/# Mole Deg F
Temperature Degrees Fahrenheit Absolute
0
500
1000
1500
2000
2500
3000
3500
4000
6
7
8
9
10
11
12
13
14
15

Fig 18

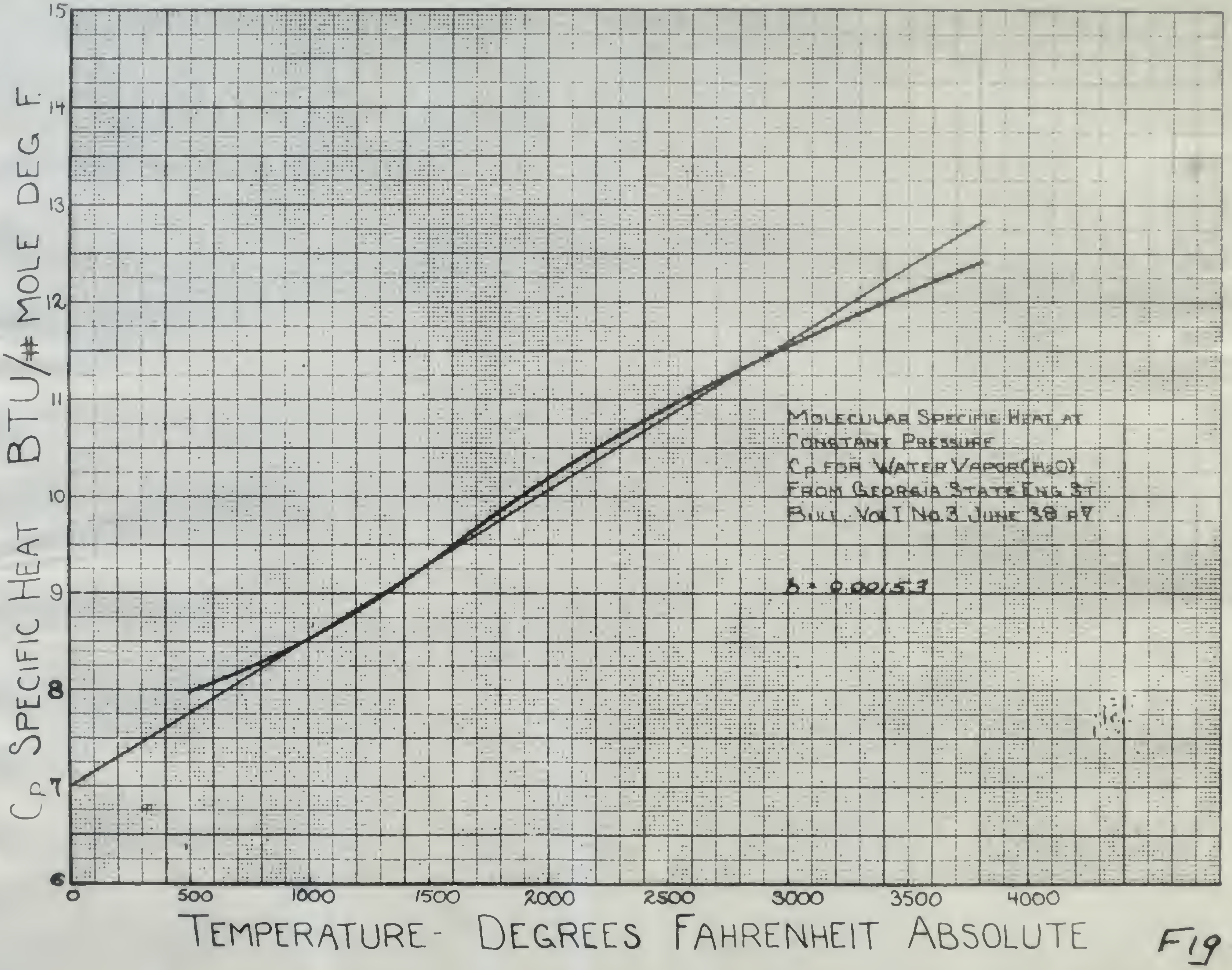
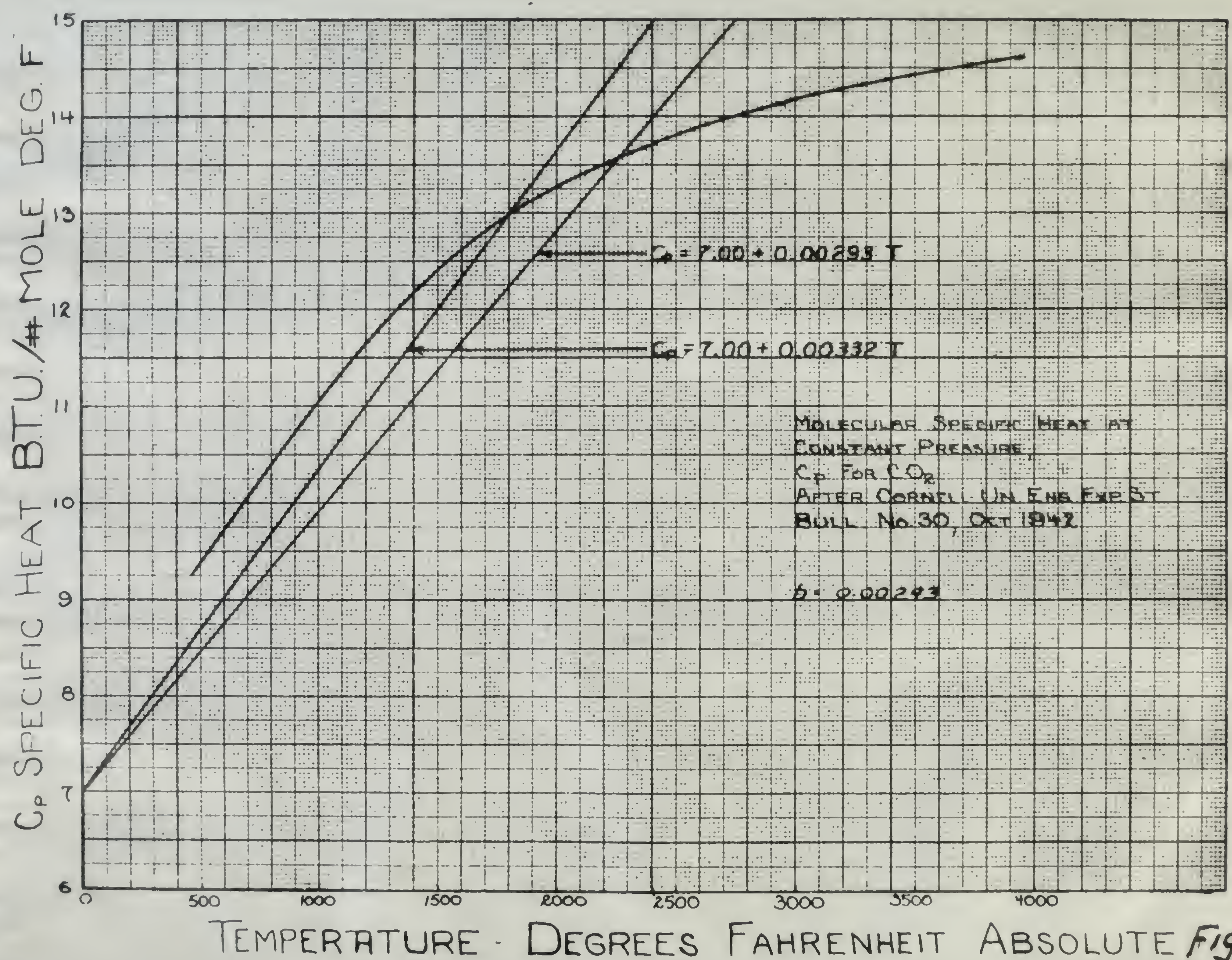

15
14
13
12
11
10
9
8
7
6
0
500
1000
1500
2000
2500
3000
3500
4000
C_p SPECIFIC HEAT BTU/# MOLE DEG F.
TEMPERATURE - DEGREES FAHRENHEIT ABSOLUTE
MOLECULAR SPECIFIC HEAT AT
CONSTANT PRESSURE
C_p FOR WATER VAPOR (H_2O)
FROM GEORGIA STATE ENG ST
BULL. VOL I NO. 3 JUNE 38 P7
b = 0.00153

Fig 19



C_p = 7.00 + 0.00293 T
C_p = 7.00 + 0.00332 T
Molecular Specific Heat at
Constant Pressure,
C_p for CO_2
After Cornell Un Eng Exp St
Bull No 30, Oct 1942
b = 0.00293
C_p Specific Heat B.T.U./# Mole Deg. F
Temperature - Degrees Fahrenheit Absolute
0
500
1000
1500
2000
2500
3000
3500
4000
6
7
8
9
10
11
12
13
14
15

Fig. 20